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First award 1950 to H. R. Mill (see page 3.6)

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On the theory of large-scale vortex motion in the atmosphere

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(Manuscript received 11 February 1949, revised 20 September 1949)

SUMMARY

A consideration of the parameters best suited to characterising a circular vortex leads to the conclusion that an intensity factor, the central pressure defect (or excess) and a scale, or size factor are fundamental.

General expressions are given for the mass defect of a cyclone, its total potential energy, and the total kinetic energy of geostrophic winds. These parameters are estimated for 17 cyclones on the basis of an assumed pressure-profile shape. It is shown that the ratio of the mean geostrophic kinetic energy to the mean potential energy is of the order 0·08.

A dimensionless measure of the 'flexure' or bending of a pressure profile is shown to vary relatively slightly from cyclone to cyclone, suggesting that the profile shape is roughly the same for all symmetrical lows.

The kinetic energy arising from ageostrophic components of wind are also discussed and it is shown that a quasi-geostrophic vortex in bodily movement has the same (ageostrophic) kinetic energy as a rigid body moving with the same speed and having a specified mass, the virtual mass of the vortex. It is shown that the virtual mass of a cyclone is of the same order as the total mass defect.

It is shown how elleptical vortices, troughs and wedges may be treated, and a 'front' is shown formally to be representable as a line distribution of circular vortices.

On the basis of the models introduced, a dynamic theory of quasi-geostrophic vortex interaction is put forward, based on the hypothesis that in mutual interaction the pressure profiles of individual vortices are conserved.

This theory leads to the conception of a potential barrier inhibiting the over-close approach of two vortices, and permits vortex interaction to be conceived as the motion of attracting and repelling 'centres of action.'

INTRODUCTION

Before 1919 the classical theory of weather development, associated mainly with the vagaries of nearly radially symmetrical vortices, was never seriously challenged. In that year the Norwegian School drew attention to the importance of air-mass discontinuities in the weather process. The new theory gave such improved leads to the short-term weather process that the front quickly displaced the vortex as the unit of meteorological thought.

As a consequence, interest in rectilinear vortex theory is practically dead, and the old model of the circularly symmetrical vortex is safely embalmed in the text-books. Nevertheless, the closed circulation with approximate thermal symmetry *is* a feature of the weather map, whether we are interested or no. The wave theory of depressions only applies satisfactorily in the early stages of cyclogenesis ; it can make no pronouncement on the structure and intensity of the finally occluded depression because its mathematical foundation, linear perturbation theory, is no longer appropriate at this stage.

The theory of long waves in steady zonal flow also depends on linear perturbation theory, and to this extent cannot be applied to large perturbations, such as are associated with closed circulations.

It is the purpose of this paper to examine the older models of the symmetrical vortex, in an attempt to ascertain why such models have proved an unsatisfactory guide in general forecasting, and to examine models which are free from the defects revealed.

A successful representation of cyclonic structure by a model involves a prior discussion of the parameters which may be used most conveniently to describe vortex properties.

Pre-frontal theories sought to account for the motion of closed pressure-systems on the basis of the Helmholtz theory of plane vortex interaction (Lamb 1932)*. Helmholtz first enunciated the important theorem that the circulation in any circuit moving in a non-viscous, barotropic fluid is constant in time. As a corollary, a vortex filament embedded in an irrotational fluid moves with the fluid. This provides a satisfactory kinematic basis for the motion of vortices small compared with the dimensions of the general flow field.

Meteorologists have long held that a ' secondary ' low will tend to rotate about its primary. This may be regarded as an application of the result in plane vortex theory, that two vortices of like sign will rotate about their common ' centre of gravity ' (Lamb 1932). If a vortex is embedded in a constant wind field it will move in the direction of the general wind, and the maximum wind around it will have the same direction owing to local mutual reinforcement of the two systems. This is the kinematic basis of the forecasting rule that a closed pressure system tends to move in the direction of the strongest wind about it. In the atmosphere it is not always possible to identify a general circulation of dimensions large compared with those of individual vortices, and for this reason the prognostications of the kinematic theory are valuable only in the case of disturbances of small dimensions, such as tropical cyclones, and extratropical ' secondaries.'

Kinematical theory can give no guide to the mutual interactions of vortices of comparable size ; for this problem we require a dynamical theory of vortex interaction. An attempt will be made to consider what necessary conditions a vortex must fulfil in order to provide a suitable model for dynamical interaction theory.

In the following the term vortex will be taken to apply to any circularly symmetrical wind field, whether cyclonic or anticyclonic.

2. The parameters of a vortex

The properties of atmospheric vortices have hitherto only been specified in a vague qualitative sense. Little attention has been devoted to the question of how best to classify vortices quantitatively, but a satisfactory classification in quanti-

* Rayleigh, Shaw, Brunt and Goldie, amongst many others have made important contributions to vortex theory as applied to meteorology. For a complete bibliography see Goldie (1937).

tative categories is a necessary prelude to a discussion of vortex models.

Two properties of a vortex call for specification, the wind field and the scalar field of pressure associated with it. The wind field of a steady state, frictionless, circular vortex with no vertical motion, is uniquely specified by the pressure-profile, and *vice-versa*. Both fields are specified immediately one of them is, so that the vortex is completely specified once the wind field, or the pressure field is given, the field chosen for a ' primary ' specification of the vortex being a matter of no theoretical concern.

We propose to describe vortices in terms of pressure profiles, as it is normally the practice in mathematical physics to treat the scalar field as ' fundamental ' and the vector field as a ' derived ' entity. This accords also with practice, for the synoptic chart of pressure is normally taken as a basis for analysis and forecasting.

The pressure profile at fixed height only defines the vortex for one level, and a complete three-dimensional specification must include a statement of how the profile parameters vary with height.

The salient properties of a pressure profile may be represented in simplest terms by two parameters ; one, an intensity factor, must measure in some way the excess or defect of pressure in the vortex area ; the other, a scale factor, represents the size of the pressure system.

We define the *intensity*, h, of a vortex as the difference between the pressure, p, at the centre, and at very great distance therefrom (p_0).

A convenient measure of size is the *spread radius*, R, or, simply, spread, defined as the distance from the centre at which the gradient of pressure becomes a maximum. The spread is the point of inflection on the pressure profile (Fig. 1).

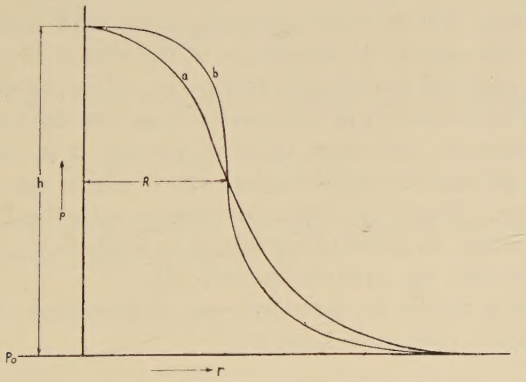

Figure 1. Pressure profiles.

It is obvious that a two-parameter specification gives only the broad features of a pressure profile. The two curves of Fig. 1 have the same intensity and spread, but they are nevertheless very different, curve b being more flexed, or bent, than curve a.

A parameter describing this property of the curve might be called the *flexure*.

The flexure is great when the pressure-gradient at R is high compared with h/R so that we might adopt as a dimensionless measure of flexure, G, the ratio

$$G = \left(\frac{\partial p}{\partial r}\right)_R \Big/ \frac{h}{R}, \text{ or } \rho_R \frac{\lambda R V_m}{h} = G \quad . \quad . \quad . \quad (1)$$

where λ is the Coriolis parameter $2\ \omega \sin \phi$, ρ_R the density of air at R, and V_m the maximum geostrophic wind speed.

Another parameter is required to specify the vertical extent of the vortex. We define the *equivalent height* Z_E by the relation

$$h_0 Z_E = \int_0^\infty h\, dz$$

h_0 being the surface value of the intensity. For reasons which will appear later we also define the *equivalent kinetic height*, Z_K, by

$$h_0^2 Z_K/\rho_0 = \int_0^\infty \frac{h^2\, dz}{\rho_0} \quad . \quad . \quad . \quad . \quad (2)$$

When the intensity varies with height according to the relation

$$h/h_0 = \rho/\rho_0$$

the two equivalent heights are identical. This type of variation of intensity with height implies a geostrophic wind speed unvarying with height, *i.e.*, a barotropic vortex. Hence in the case of a barotropic vortex, Z_E and Z_K are identical and equal to $p_0/g\rho_0$.

The tilt (slope and direction) of the axis of the vortex is also required for a complete specification, but in most applications we shall assume this axis to be vertical. Finally, the velocity of translation of the vortex as a whole*, C, and its acceleration $\dot{C}$ are required to define its motion as well as its structure.

We see that a large number of parameters enter into the specification of vortex structure and movement. In other fields of physics it is frequently fruitful to derive from such parameters dimensionless ratios which may be given a simple physical significance. There are three parameters with the dimensions of speed associated with a vortex, its speed of translation as a whole, C, the maximum wind-speed within the vortex, V_m, and the quantity λR.

From these three speeds we can derive two independent, dimensionless ratios. The *Critical Rotation Ratio* is defined as

$$\alpha = V_m/(\lambda R) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The rotation ratio $V/\lambda r$, where V is the wind-speed at a distance r from the centre of the vortex, is a measure of the ratio between the cyclostrophic and geostrophic components of wind, α measuring this ratio in the zone of maximum wind speed. When α is small winds are approximately geostrophic throughout, so that we may define a *geostrophic vortex* as one with vanishingly small α.

* The meaning of the translation velocity for a pressure system as a whole has been discussed by James (1948).

The *translation ratio*, β, is defined by

$$\beta = C^2/\lambda^2 R^2$$

In addition to the speed parameters there is a set of seven parameters with the dimensions of pressure, the three most important of which are h, $\frac{1}{8}\rho_R \lambda^2 R^2$, $\rho_R \lambda R V_m$, where ρ_R is the density of air at the spread radius. Since density varies only slightly in the horizontal plane we shall normally drop the suffix, but it is to be understood that where ρ enters into a dimensionless parameter, the density at distance R from the centre is implied.

From these three it is possible to derive two independent dimensionless parameters, the *vortex number*, Q, defined by

$$Q = 8h/\rho\lambda^2 R^2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the flexure

$$G = \rho\lambda R V_m/h \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which has already been defined.

All these parameters are kinematic in content. There are other parameters describing the overall properties of a vortex, which have dynamic implications. First of these is the total kinetic energy of the vortex.

$$K = \tfrac{1}{2} \int \rho V^2 \, d\sigma \, dz \quad . \quad . \quad . \quad . \quad (6)$$

where the integration is extended over the whole of space occupied by the vortex.

We may define a total heat content for the volume occupied by the vortex

$$c_v \int T\rho \, d\sigma \, dz$$

c_v being the specific heat of air at constant volume, and a total gravitational energy

$$\int \rho g z \, d\sigma \, dz$$

The sum of these two quantities, the total *potential* energy of the volume of air within the vortex is easily shown to be

$$\frac{c_p}{k} \int p \, d\sigma \, dz$$

where c_p is the specific heat at constant pressure, and k the gas constant.

Not all this potential energy belongs to the vortex as such for in its absence we should have a field of pressure, p_0, uniform in the horizontal plane, with an associated potential energy

$$\frac{c_p}{k} \int p_0 \, d\sigma \, dz$$

Thus the total potential energy of the vortex is the difference between these quantities, or,

$$P = \frac{c_p}{k} \int (p - p_0)\, d\sigma\, dz \quad . \quad . \quad . \quad (7)$$

The potential energy is negative for a cyclone, and positive for an anticyclone.

Closely allied to the potential energy is a surface integral

$$W = \int |p - p_0|\, d\sigma \quad . \quad . \quad . \quad . \quad (8)$$

which has been termed the *strength* of the vortex (James 1948), the integration being extended over the surface of the earth covered by the vortex. It represents the total weight of air which must be withdrawn from (or added to) a uniform field of pressure to form a cyclone (or anticyclone) divided by the acceleration of gravity.

Vortex models are physically acceptable only if their strength, total potential and total kinetic energies are all finite.

3. The Rankine vortex

The Rankine combined vortex is defined by the tangential wind-field

$$V = V_m \,.\, r/R \text{ for } r < R,\ V = V_m \,.\, R/r \text{ for } r > R \quad . \quad . \quad (9)$$

The spread radius thus separates an inner core in solid rotation from an outer, irrotational flow field.

This type of wind distribution has long been regarded as typical of atmospheric vortices, and Goldie (1937, 1939) finds that it does provide a good fit to a number of extratropical depressions.

However, there are grave difficulties in the way of accepting the Rankine vortex as a model for atmospheric vortices. The kinetic energy of a Rankine vortex of unit thickness (height) is easily shown to be

$$K = \tfrac{1}{4}\, \pi\, \rho\, R^2 + \pi\, R^2\, \rho \int_R^\infty \frac{dr}{r} \,.$$

The integral is infinite, so that the total kinetic energy of a Rankine vortex is infinite. The only way to avoid this difficulty is to assume that the rotational field ceases at some distance S from the centre, in which case we should have

$$K = \tfrac{1}{4}\, \pi\, \rho\, R^2 + \pi\, R^2\, \rho\, \log_e (S/R)$$

Nothing corresponding to a sudden cessation of wind at a fixed distance has been observed in connection with vortices in the atmosphere, and the value of K depends strongly on S, something which has not been observed, and probably does not exist. The circumference, S, would constitute a ring of vorticity, and therefore this solution robs the Rankine vortex of the virtue of mathematical simplicity arising from the fact that motion is irrotational everywhere outside the central core.

The pressure profile consistent with geostrophic winds in a Rankine vortex is such that pressure becomes indefinitely high remote from the centre of a cyclone, and infinite and negative remote from an anticyclone. The intensity, strength and total potential energy are all infinite for a true Rankine vortex.

Since the Rankine vortex cannot be used in dynamical reasoning, it must be abandoned as a model for the symmetrical atmospheric vortex. It is perhaps not surprising that a model of the atmospheric vortex subject to such grave drawbacks should have proved disappointing in practical application.

4. The general circular vortex

If the Rankine vortex is an unsatisfactory model, we must select a pressure profile with the requisite convergence properties. We take as our form of pressure profile

$$p = p_0 + hf(r/R) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where we have two parameters, the intensity h, and the spread R. The profile function f has the properties that $f(0) = \pm 1$, the positive sign applying to anticyclones, $f(\infty) = 0$, and f' is a maximum for $r = R$, by definition of the spread.

The geostrophic (tangential) wind speed in such a vortex is

$$V = \left|\frac{1}{\rho\lambda}\frac{\partial p}{\partial r}\right| = \left|\frac{h}{\rho\lambda R} f'\right| \quad . \quad . \quad . \quad . \quad (11)$$

which has the maximum value at $r = R$

$$V_m = \frac{h}{\rho\lambda R} |f'(1)|$$

The flexure is thus

$$G = \rho\lambda R V_m/h = |f'(1)|$$

The strength, from Eq. (7), is

$$W = 2\pi h_0 R^2 \int_0^\infty f(s)\, s\, ds \quad (s = r/R) \quad . \quad . \quad (12)$$

and the total potential energy

$$= 2\pi c_p h_0 R^2 Z_E \int_0^\infty f(s)\, s\, ds/k \quad . \quad . \quad (13)$$

when R is independent of height. In order that the strength and potential energy should be finite we must have f decreasing more rapidly than $1/s^2$ for large s. This imposes a restriction on the form of the profile, which, we have seen, is not satisfied by the Rankine vortex model.

The geostrophic kinetic energy is

$$K = \pi \int_0^\infty \int_0^\infty \rho V^2 r\, dr\, dz = \frac{\pi h_0^2 Z_K}{\rho_0 \lambda^2} \int_0^\infty [f']^2\, s\, ds \quad . \quad . \quad (14)$$

and the ratio of kinetic to potential energy

$$K/P = \frac{k Z_K}{c_p Z_E} \cdot \frac{h_0}{2\rho_0 \lambda^2 R^2} \frac{\int_0^\infty [f']^2 s\, ds}{\int_0^\infty f\, s\, ds} = \frac{Q Z_K}{56 Z_E} \cdot \frac{\int_0^\infty (f')^2 s\, ds}{\int_0^\infty f\, s\, ds} \quad . \quad . \quad (15)$$

The ratio of kinetic to potential energy thus depends on the vortex number Q and the form of the pressure profile. It will be borne in mind that the ratio of geostrophic kinetic to potential energy is only an approximation to the ratio of kinetic to potential energy, for winds are normally sub-geostrophic in a cyclone, and super-geostrophic in an anticyclone.

Explicit values of G, W, P and K may now be derived for a circular system if the form of the pressure-profile function f is known. We consider two forms which roughly represent observed pressure profiles.

A *sinoidal vortex* is defined by the profile function

$$f = \frac{1 + \cos(\pi r/2R)}{2} \text{ where } r \leq 2R, \text{ and } f = 0 \text{ for } r \geq 2R.$$

For this type of profile we have,

$$G = |f'(1)| = \pi/4 = 0{\cdot}785$$

$$W = 2\pi h_0 R^2 (1 - 4/\pi^2) = 1{\cdot}19\,\pi h_0 R^2$$

$$K = (\tfrac{3}{4} - 2/\pi^2)\,\pi h_0^2 Z_K/(\rho_0 \lambda^2) = \pi h_0^2 Z_K/(1{\cdot}81\,\rho_0 \lambda^2)$$

$$K/P = QZ_K/64Z_E$$

The *normal vortex* is defined by,

$$f = e^{-r^2/2R^2}$$

For a normal vortex we have

$$G = e^{-\frac{1}{2}} = 0{\cdot}61$$

$$W = 2\pi h_0 R^2$$

$$K = \pi h_0^2 Z_K/(2\,\rho_0 \lambda^2)$$

$$K/P = QZ_K/112\,Z_E$$

The flexure is a function of the type of pressure profile. Hence measurements of this parameter establish the type of profile appropriate to the vortex studied.

5. The flexure

We have seen that the flexure is the same for all normal geostrophic vortices. If we can determine the magnitude of this parameter for atmospheric vortices we shall have a measure of the degree to which actual atmospheric profiles are approximated by the normal form.

The values of all three parameters entering into the flexure are subject to considerable uncertainties, but we are fortunate in having a full documentation of 20 extra-tropical depressions by Goldie (1939), who gives estimates of R, central pressure and maximum wind speed.

Goldie selected for study mainly intense, mature depressions although complete thermal symmetry is by no means a feature of all the cases studied. However, we may take the figures as broadly applicable to intense, occluded depressions.

TABLE I. CYCLONE PARAMETERS

(1) No.	(2) R (km)	(3) V (m/sec)	(4) V_m (m/sec)	(5) $\alpha = V_m/\lambda R$	(6) h (mb)	(7) G	(8) M[gm x 10^{17}]	(9) P[ergs x 10^{26}]	(10) K[ergs x 10^{26}]	(11) Q	(12) M_v [gm x 10^{17}]
1	550	19·6	25·0	0·28	43	0·53	8·2	16·9	1·04	5·5	4·3
2	500	19·5	25·5	0·31	48	·44	7·7	15·8	1·29	7·4	6·4
3	450	19·5	26·2	0·34	43	·45	5·6	11·6	1·04	8·2	6·4
4	280	29·0	55·8	0·82	52	·47	2·6	5·3	1·52	25·5	24·5
5	420	25·0	36·6	0·47	55	·46	6·2	12·8	1·70	12·0	12·0
6	450	26·6	39·2	0·47	61	·46	7·9	16·3	2·09	11·6	12·9
8	360	18	25·2	0·40	21	·71	1·3	2·7	0·25	6·3	2·4
10	380	20	28·3	0·42	21	·83	1·9	3·8	0·25	5·6	2·2
11	470	18	23·6	0·31	33	·63	4·7	9·7	0·61	5·8	3·5
12	480	23	15·9	0·22	25	·50	3·7	7·6	0·35	4·2	1·9
14	320	26	42·9	0·65	45	·50	3·0	6·2	1·14	17·0	14·0
15	430	27	40·5	0·50	55	·52	6·5	13·4	1·70	11·5	21·0
16	460	20	27·0	0·35	46	·49	6·3	13·0	1·19	7·5	8·0
17	500	27	38·5	0·43	63	·50	10·1	20·8	2·22	9·7	13·0
18	650	24	31·0	0·29	68	·49	18·4	38·0	2·59	6·2	13·0
19	700	23	29·0	0·26	48	·69	15·1	31·6	1·28	3·8	4·0
20	400	24	35·5	0·48	47	·50	4·8	9·9	1·24	11·3	10·0
Mean						·54	7·3	13·8	1·28	9·4	9·5

In Table I, column (1) gives a serial number attached by Goldie to each cyclone. Column (2) gives the value of R, and column (3), V, the maximum wind speed. From the maximum wind speed we can estimate the maximum geostrophic wind speed, V_m (column (4)) from the relation

$$V_m = V(1 + V/\lambda R) = V(1 + \alpha)$$

Column (5) shows the critical rotation ratio, α, and column (6) the value of h, computed as the difference between central pressure and an assumed normal value of 1013 mb. Goldie's published figures do not permit an evaluation of the asymptotic pressure value for each individual depression, but the relative error in estimating h in this way should not be large except in the case of the less intense depressions. Two of Goldie's depressions (Nos. 10 and 12) were omitted altogether from the present calculations, as the estimates of h by this method were considered too uncertain. Another depression (No. 7) was also omitted owing to the uncertainty in the estimate of R.

From columns (6), (4), and (2) we can compute the flexure (column (7)) on the assumption that 60° can be taken as the central latitude of all the depressions. It will be observed that the flexure of the 17 cyclones clusters remarkably closely around the mean value, 0·54, the values being within 30 per cent of the mean in all cases except one, No. 10. Uncertainties in the evaluation of h and R, and the fact that not all Goldie's depressions are perfectly circular, could readily account for this variation, so that the figures suggest that the flexure varies only slightly, if at all, from cyclone to cyclone. The variance in the flexure is certainly less than that of the individual parameters from which it is computed. From the quasi-invariance of the flexure we can infer that changes in profile shape from cyclone to cyclone are relatively slight.

The mean value of the flexure, 0·54, is slightly lower than the value of this parameter in the normal vortex, 0·61, and appreciably lower than the flexure of the sinoidal vortex, 0·785. From this it is possible to assert that the normal vortex is a better model for an atmospheric cyclone than the sinoidal vortex.

The importance of the flexure in synoptic meteorology has yet to be determined. The table shows that it varies only within narrow limits, but whether it is a true constant for extratropical vortices is a question for further research.

Column (8) shows the mass defect, M, of the depressions, computed on the assumption that pressure profiles are normal. The mean mass defect is $7{\cdot}3.10^{17}$ gm or $7{\cdot}3.10^{11}$ metric tons, which is of the order of magnitude quoted by Shaw (1932) for European depressions. It will be noted that the extremes of mass in the table are in the ratio 14:1. It would seem that depressions do not show a very wide variation in mass, although it must be remembered that the bias is very definitely towards intense depressions in the Goldie series. Small ' secondaries ' may have a mass defect very much less than the quoted average figure. Shaw (1932) quotes for the mass of a small secondary over Northern Europe in July-August 1917, 7.10^{10} tons, or about 1/10th of the mean mass of the table. The mass multiplied by the acceleration of gravity gives the strength. The mean strength of the lows was thus of the order 7.10^{20} gm weight. The strength multiplied by $c_p Z_E/k$ gives the total potential energy. This quantity, computed on the assumption that $Z_E = 6000$ m in each case, is shown in column (9) in units of 10^{26} ergs.

Column (10) shows the total geostrophic kinetic energy of the depressions in units of 10^{26} ergs. The mean kinetic energy is $1{\cdot}28.10^{26}$ ergs, with extreme variations in the ratio 10:1. Variations in the kinetic energy of cyclones are of the same order as variations in the potential energy.

The mean geostrophic kinetic energy is slightly over 8 per cent of the mean potential energy. This is an overestimate of the ratio of actual kinetic energy to potential energy, for winds are sub-geostrophic in a cyclone.

Column (11) shows the vortex number of depressions. The mean number is 9·4, with extreme variations of 7 to 1. The ratio of mean geostrophic kinetic energy to mean potential energy is given by

$$\overline{K}/\overline{P} = \frac{\overline{Q}\ Z_K}{112 Z_E}$$

and agrees with the value computed directly from columns (9) and (10). The values of potential energy and kinetic energy have been based on the assumption that $Z_K = Z_E$, *i.e.*, the vortices were assumed to be barotropic. In general $Z_K < Z_E$, so that the ratio of geostrophic kinetic energy is less than the 8 per cent quoted above.

6. The power vortex

We have seen that the normal profile calls for a slightly higher flexure than is observed in actual atmospheric vortices. To get a closer representation to vortex profiles one might try a three parameter representation. A suitable form is

$$p = p_0 + \frac{h}{1 + \frac{n-1}{n+1}\left(\frac{r}{R}\right)^n} \quad . \quad . \quad . \quad (16)$$

which we term a *power* profile (Bjerknes, *etc.* 1934). The new parameter n may be termed the *index* of the profile. We shall only quote the flexure of such a profile, which is

$$G = (n^2 - 1)/4n \quad . \quad . \quad . \quad . \quad . \quad (17)$$

G is a function of the index only in the power profile, high values of flexure being associated with high values of n. An index of 2·55 gives a G of 0·54, the mean value found for 17 cyclones. The profile

$$p = p_0 + \frac{h}{1 + \frac{1{\cdot}55}{3{\cdot}55}\left(\frac{r}{R}\right)^{2{\cdot}55}}$$

thus gives a better fit to cyclones than the normal profile. However, the accuracy of the data at present available scarcely warrants the use of elaborate profiles. That the power profile should provide a better fit than the normal profile is not necessarily physically significant, for a mathematical fit may be improved indefinitely by increasing the number of arbitrary parameters.

Consequently we shall adopt the normal profile as our atmospheric model, while remarking that the results obtained by its use are qualitatively valid also for other profiles.

7. The isallobaric energy term

In the table we have given estimates of the kinetic and potential energies of 17 depressions, with the assumption that the pressure profile is normal, and the winds are geostrophic. In the atmosphere winds are only approximately geostrophic, so that in the energy economy of a vortex we must consider other terms connected with the departure of wind speed from its geostrophic value.

As is well known, curvature of path reduces the wind speed in cyclonic motion, and increases it in anticyclonic motion. Associated with this departure of speed from its geostrophic value we may think of a vortex possessing a total cyclostrophic kinetic energy, positive for anticyclones, negative for cyclones. The cyclostrophic kinetic energy may be of the order 1/10th or more of the geostrophic kinetic energy in cyclones of high intensity.

Another departure from the geostrophic wind is the so-called isallobaric wind called into being by a changing pressure field. The isallobaric wind blows into regions of isallobaric low, and its magnitude is given by the equation

$$V_I = \frac{1}{\rho \lambda^2} \frac{\partial^2 p}{\partial n \partial t}$$

where n is taken in the direction of the isallobaric gradient.

An isallobaric gradient can arise from two causes, from an intensifying of the vortex or a change in its size, or from its motion over the earth. We shall not consider the isallobaric gradient due to development.

A vortex may possess an isallobaric energy term due to the bodily movement of the vortex as a whole. If a normal vortex is moving along the x-axis with a constant speed, C, its pressure profile will be given by

$$p = p_0 + he^{-\frac{(x-ct)^2+y^2}{2R^2}} \quad . \quad . \quad . \quad (18)$$

Hence

$$\left(\frac{\partial p}{\partial t}\right)_{t=0} = \frac{hCx}{R^2} \cdot e^{-\frac{x^2+y^2}{2R^2}} \quad . \quad . \quad . \quad (19)$$

The isallobaric wind is therefore

$$= \frac{h\ C\ e^{-r^2/2R^2}}{\rho\ \lambda^2\ R^2} \left[\mathbf{i}\ (1 - x^2/R^2) - \mathbf{j}\ x\ y/R^2\right] \quad . \quad . \quad (20)$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the direction of x and y respectively.

The kinetic energy of a moving geostrophic vortex is now

$$\tfrac{1}{2}\rho\,(V_g + V_I)^2 = \tfrac{1}{2}\rho\,V_g^2 + \rho\,V_g\,V_I + \tfrac{1}{2}\rho\,V_I^2$$

Integrated throughout the vortex the cross-term is zero. The additional kinetic energy due to the motion is thus

$$\tfrac{1}{2}\rho\,V_I^2$$

Such an integration performed over a disc of unit thickness is found to be

$$k_I = \frac{C^2\,h^2}{2\rho\,\lambda^4\,R^4} = \frac{C^2}{\lambda^2\,R^2}\,k_g \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Integrating with respect to height we have the expression for the total isallobaric kinetic energy in terms of the total geostrophic kinetic energy

$$K_I = \frac{C^2}{\lambda^2\,R^2}\,K_g$$

The total energy of the vortex is

$$E = K_I + K_g = K_g\left(1 + \frac{C^2}{\lambda^2\,R^2}\right) \quad . \quad . \quad . \quad (22)$$

Although this expression has only been established for a normal vortex, it is easy to show that it holds generally for any profile-shape.

The expression for the isallobaric energy is only an approximation, valid when this energy is a small proportion of the total energy, *i.e.*, when the translation ratio $\beta = C^2/\lambda^2\,R^2$ is small. This condition is nearly always satisfied for actual vortices. In the Goldie series the highest value found is $\beta = 0{\cdot}185$ for cyclone No. 8. The

geostrophic kinetic energy might be called the internal energy of the vortex, as it depends on the structure of the vortex as such, while the designation of external energy for the isallobaric kinetic energy indicates that it depends on the speed of the vortex as a whole through the ambient fluid.

By introducing the new parameter $M_V = 2\ K_g/\lambda^2 R^2$, we can write for the external, or isallobaric energy

$$K_I = \tfrac{1}{2} M_V C^2 \qquad . \quad . \quad . \quad . \quad . \quad (23)$$

i.e., a vortex moving with speed C has the same external kinetic energy as a rigid body of mass M_V moving with the same speed. We may term M_V the virtual mass of a vortex.

The virtual mass of a normal vortex is

$$M_V = (\pi\ \rho\ Q^2\ R^2\ Z_K)/64 \qquad . \quad . \quad . \quad (24)$$

where Q is the vortex number, and Z_K the equivalent kinetic height. Column (12) in the table gives values of M_V computed from equation (23), on the assumption that $Z_K = 6000$ m for all the cyclones.

The average virtual mass for the 17 cyclones is $9{\cdot}5.10^{17}$ gm ($9{\cdot}5.10^{11}$ metric tons), which is of the same magnitude as the average mass of the cyclones, $7{\cdot}3.10^{17}$gm.

8. Elliptical vortices

So far we have only considered vortices with circular symmetry, but it is not difficult to obtain similar results for vortices of other shapes. We write for the pressure field in a normal, elliptical vortex,

$$p = p_0\ he^{-\frac{1}{2}(x^2/a^2 + y^2/b^2)}$$

The elliptical vortex requires two scale factors, a and b ($a > b$), the semi-major and semi-minor spreads of the vortex. An isobar is given by $x^2/a^2 + y^2/b^2 = K$, where K is a constant defining the particular isobar

$$p = p_0 + he^{-K/2}$$

The maximum wind speed on the x axis is

$$(V_y)_m = \frac{h\ e^{-\frac{1}{2}}}{\lambda\ a\ \rho} \text{ at the points } x = \pm a$$

and the maximum for the y-axis is

$$(V_x)_m = he^{-\frac{1}{2}}/\lambda\ b\ \rho \text{ at } y = \pm b$$

The absolute maximum is $(V_x)_m$ along the semi-minor axis.

We define the *principal flexures*, those for pressure profiles along the x and y axes respectively, as

$$G_x = \rho\ \lambda\ a\ (V_y)_m/h,\ G_y = \rho\ \lambda\ b\ (V_x)_m/h$$

Both the principal flexures are equal to $e^{-\frac{1}{2}} = 0{\cdot}61$, the flexure of the normal circular vortex. The circular vortex is, indeed, a special case of the elliptical vortex when $a = b = R$, the spread radius.

The potential energy of a normal elliptical vortex is

$$P = 2\pi c_p ab h_0 Z_E/k$$

while the total geostrophic kinetic energy is

$$K = \frac{\pi h_0^2 Z_K}{\rho_0 \lambda^2} \left(\frac{a}{b} + \frac{b}{a}\right)$$

When $a = b$ we recover the expressions for the normal circular vortex.

9. The line vortex

In hydrodynamics it is often convenient to treat a sheet vortex as a line distribution of infinitely small circular vortices. Thus we are able to bring fronts and troughs into the 'classical' theory by treating them as line distributions of circular vortices.

Suppose the intensity per unit length of a trough is h/R so that an element of the trough line dy may be considered as a circular vortex of intensity $h\,dy/R$. (Fig. 2). Taking the trough line along the y-axis, our problem is to find the pressure defect at a point x from the trough line, due to an element dy of the trough, and then integrate to determine the total contribution to the pressure defect at point x.

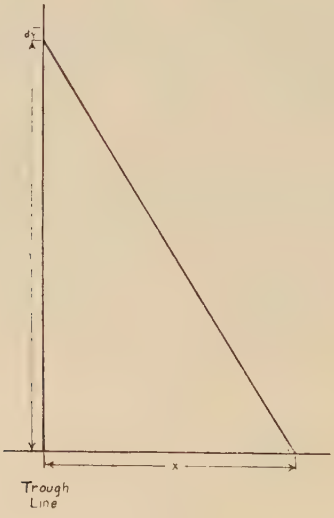

Figure 2.

The pressure defect due to the element dy is $he^{-(x^2 + y^2)/2R^2}\,dy/R$ where R is defined as the spread of the elementary vortices. The total pressure defect is

$$\frac{h}{R}\int_{-\infty}^{+\infty} e^{-(x^2 + y^2)/2R^2}\,.\,dy = \sqrt{2\pi}\,he^{-x^2/2R^2} \quad . \quad . \quad (25)$$

The strength per unit length of the trough or front is

$$\frac{h}{R}\int_{-\infty}^{\infty} e^{-x^2/2R^2}\,.\,dx = \sqrt{2\pi}\;h\;R$$

The pressure gradient in the x direction is

$\frac{\partial p}{\partial x} = -\sqrt{2\pi}\,\frac{hx}{R^2}\,e^{-x^2/2R^2}$, giving the geostrophic wind speed

$$V = \frac{\sqrt{2\pi}hx}{\rho\lambda R^2}\,e^{-x^2/2R^2}$$

along the trough line. It will be observed that V changes sign at the trough line, also that the wind in the y direction is zero there. A trough or front on a weather map normally has a pressure gradient along the trough line, but this can be neglected for present purposes because the cross terms in the kinetic energy cancel out, since the wind components are at right angles.

The kinetic energy per unit of trough line

$$\tfrac{1}{2}\int_{-\infty}^{\infty}\rho V^2\,dx = \frac{\pi\sqrt{\pi}\;h_0^{\,2}\;Z_K}{2\;\rho_0\;\lambda^2\;R} \qquad . \qquad . \qquad . \qquad (26)$$

10. The mutual interaction of vortices

The total energy of a vortex comprises numerous terms, the potential energy, the geostrophic, cyclostrophic and isallobaric kinetic energies. If two vortices are in proximity the total energy balance may become very complicated owing to mutual interaction. In a closed system the total energy must remain constant, so that any loss, say, in the field of potential energy must re-appear in one or other of the kinetic energy terms.

The concept of a closed system implies constancy of mass as well as constancy of energy. Hence we shall require that the total mass defect (or excess) due to two vortices is the sum of the individual mass defects (or excesses).

It will be assumed that in vortex interaction the pressure profiles of the individual vortices are preserved, so that the combined pressure field is determined by the superposition of the constituent fields. This we treat as a pure hypothesis, to be tested by applying its consequences to the observed interaction of pressure systems. It is applied subconsciously in meteorology, for a forecaster has an instinctive feeling for the integrity of a vortex, as expressed by its pressure field.

The hypothesis of additive pressure fields satisfies the requirements for a closed system, for if pressure fields are additive, so are total mass defects or excesses. If the equivalent heights of the vortices remain unchanged in interaction, potential energy, being proportional to mass excess or defect, is likewise conserved.

A closed system is possible in which pressure fields are not additive, if, for example, there is a transfer of mass from one vortex to the other. This, however, must be regarded as an example of 'development' within the pressure field. Pressure fields will only be conserved in interaction when there is no development

so that the hypothesis of additive pressure fields is equivalent to the assumption that there is no development in the field of pressure.

A difficulty is encountered when the energy balance is considered. Suppose two identical, geostrophic vortices are superposed, the pressure fields being taken as additive. Then the field of the fused vortices is represented by a new vortex with the same spread and twice the intensity. The geostrophic kinetic energy varies with the square of the intensity, so that the combined vortex will have four times the kinetic energy of a constituent vortex. The potential energy is unchanged, hence the total energy of the combined vortices exceeds that of the two systems in isolation. The conditions for a closed system are not satisfied, hence stationary geostrophic vortices could never fuse unless energy is imparted from outside, or there is a transfer between potential and kinetic energy within the system, *i.e.*, unless there is development.

If, however, one of the vortices were in motion, its isallobaric kinetic energy could become available, through the slowing down of the system, to provide the geostrophic energy necessary to provide a fusion within a closed system.

It is thus possible to develop a consistent dynamical theory of vortex interaction under conditions of no development in closed systems, by considering the interchange between isallobaric and geostrophic kinetic energies. It will only be possible to apply such a theory to the interaction of atmospheric vortices so long as the cyclostrophic kinetic energy is only a small proportion of the geostrophic energy. This confines strict application to vortices of low intensity and large spread.

11. The mutual energy of adjacent vortices

It is the purpose of this section to consider the wind field and the resultant kinetic energy due to two adjacent vortices ; geostrophic winds are assumed.

We consider a normal vortex of intensity h_1, spread R_1, centred at the point (o, o) and one of intensity h_2, spread R_2, centred at (x_0 , o).

The geostrophic wind speed in the x-direction is

$$\left.\begin{aligned} U = \frac{1}{\rho\lambda}\frac{\partial p}{\partial y} &= \frac{h_1 y}{\rho\lambda R_1^{\,2}} e^{-(x^2+y^2)/2R_1^2} + \frac{h_2 y}{\rho\lambda R_2^{\,2}} e^{-[(x-x_0)^2+y^2]/2R_2^2} \\ \text{and in the y-direction} & \\ V = \frac{1}{\rho\lambda}\frac{\partial p}{\partial x} &= \frac{h_1 x}{\rho\lambda R_1^{\,2}} e^{-(x^2+y^2)/2R_1^2} + \frac{h\,(x_2 - x_0)}{\rho\lambda R_2^{\,2}} e^{-[(x-x_0)^2+y^2]/2R_2^2} \end{aligned}\right\} \quad (27)$$

The kinetic energy of the wind at any point is

$$\left.\begin{aligned} \tfrac{1}{2}\rho(u^2+v^2) = \frac{1}{2\rho\lambda^2}\Bigg[& \frac{h_1^{\,2}(x^2+y^2)}{R_1^{\,4}} e^{-(x^2+y^2)/R_1^2} + \frac{2h_1 h_2}{R_1^{\,2}R_2^{\,2}}(x^2 - xx_0 + y^2) \\ & e^{-\left\{\frac{x^2+y^2}{2R_1^2} + \frac{(x-x_0)^2+y^2}{2R_2^2}\right\}} + \left\{(x-x_0)^2 + y^2\right\}\frac{h_2^{\,2}}{R_2^{\,4}} e^{-\left\{\frac{(x-x_0)^2+y^2}{R_2^2}\right\}}\Bigg] \end{aligned}\right\} \quad (28)$$

The first and last terms are the geostrophic or inner kinetic energies due to vortices 1 and 2 respectively. The middle term is new, and represents the kinetic energy due to the interaction of the two vortices. This term, which may be called the mutual energy of the vortices, we shall proceed to evaluate.

Writing $2/R^2 = 1/R_1^2 + 1/R_2^2$, $x' = x - R^2 x_0/2R_2^2$, $(\gamma')^2 = (x')^2 + y^2$ we have for the mutual energy at a point

$$k_{12} = \frac{4 h_1 h_2}{\rho\lambda R_1^2 R_2^2}\left[(\gamma')^2 + R^2 + x' x_0 (R_2^2/2 - R_1^2/2) - R^2 x_0^2/4 R_1^2 R_2^2\right] e^{-\left\{\frac{(\gamma')^2}{R^2} + \frac{x_0^2 R^2}{4 R_1^2 R_2^2}\right\}} \quad . \quad (29)$$

The mutual energy for a disc of unit thickness is by integration

$$k_{12} = \frac{4\pi h_1 h_2 R_1^2 R_2^2}{\rho\lambda^2 (R_1^2 + R_2^2)^2}\left[1 - x_0^2/2 (R_1^2 + R_2^2)\right] e^{-x_0^2/2 (R_1^2 + R_2^2)}$$

To get the total mutual kinetic energy we integrate with respect to height. Defining a mutual kinetic height

$$\frac{h_{01} h_{02} Z_m}{\rho_0} = \int_0^\infty \frac{h_1 h_2}{\rho} dz$$

we have for the total mutual kinetic energy

$$K_{12} = \frac{4\pi h_{01} h_{02} R_1^2 R_2^2 Z_m}{\rho\lambda^2 (R_1^2 + R_2^2)^2}\left[1 - x_0^2/2 (R_1^2 + R_2^2)\right] e^{-x_0^2/2 (R_1^2 + R_2^2)} \quad . \quad . \quad (30)$$

the suffix 0 referring to surface values.

We shall assume in the following that both vortices have the same kinetic height, Z, which is equal to the mutual kinetic height, Z_m. Under these conditions we may express the mutual kinetic energy as a function of the inner kinetic energies of the two vortices, and their separation, x_0,

$$K_{12} = \pm 8\sqrt{K_1 K_2}\, \frac{R_1^2 R_2^2}{(R_1^2 + R_2^2)^2}\left[1 - x_0^2/2(R_1^2 + R_2^2)\right] e^{-x_0^2/2(R_1^2 + R_2^2)} \quad . \quad . \quad (31)$$

The sign to be taken is positive for like vortices, negative for unlike vortices.

The equation (31) embodies all the essential facts concerning the mutual interaction of normal, geostrophic vortices, and in the next section it will be discussed in detail. Fig. 3 shows the mutual energy curve for two like, normal vortices of identical strength and spread.

12. The barrier effect

The mutual kinetic energy of two like vortices has from (31), a maximum value,

$$K_{max} = 8\sqrt{K_1 K_2}\,\frac{R_1^2 R_2^2}{(R_1^2 + R_2^2)^2}$$

if $x_0 = 0$, and a minimum

$$K_{min} = -8\sqrt{K_1 K_2}\,\frac{R_1^2 R_2^2}{(R_1^2 + R_2^2)^2} \cdot e^{-2}$$

at $x_0 = 2$.

These extremes depend only on K_1, K_2, and the ratio of R_1 to R_2.

The mutual energy is zero for $x_0 = \sqrt{2}\,(R_1^2 + R_2^2)$ and $x_0 \longrightarrow \infty$. It is positive in the range 0, $\sqrt{2}\,(R_1^2 + R_2^2)^{\frac{1}{2}}$ of x_0, and negative outside this range.

For unlike vortices the ranges of positive and negative mutual energy are interchanged.

For two vortices of given inner energies, the maximum and minimum of the mutual energy curve reaches the largest numerical values when the spreads are equal, or $R_1 = R_2$. In this case the maximum and minimum of the curve are respectively $2\sqrt{K_1 K_2}$ and $-2e^{-2}\sqrt{K_1 K_2}$.

For vortices of equal inner energies and equal spreads the maximum mutual energy is twice the inner energy of an individual vortex. It follows that two identical, coincident vortices are the same in every respect as a single vortex of twice the intensity and four times the inner energy. In the case of unlike vortices of equal intensity and spread, coincidence implies annihilation, the negative mutual energy just balancing the inner energies of the two vortices.

If the spread of one vortex is much greater than the spread of the other, say $R_1 >> R_2$, the maximum mutual energy is, approximately

$$K_{max} = 8\sqrt{K_1 K_2}\,R_2^2/R_1^2$$

Thus the smallest mutual energies are associated with vortices of widely differing spreads.

The mutual kinetic energy of adjacent vortices is a new term in the energy balance of a vortex system. We have seen that a vortex possesses a number of energy terms — the net potential energy, the geostrophic kinetic energy, the kinetic energy due to ageostrophic components of wind, such as the isallobaric energy of movement. In considering a closed system of two or more vortices, the total energy must remain constant, so that an increase in the mutual energy of the system can only take place at the expense of the energy terms just enumerated.

If the system is not closed the mutual energy may also increase at the expense of an outside energy source, by radiant or latent heat exchanges, *etc.* Such changes are important, and the ultimate object of the energy approach to vortex theory is to assess their influence on the history of vortex development.

The most immediate programme is that of studying the simplest possible case of vortex interaction in a closed system in which the potential and geostrophic kinetic energies are invariant, and the ageostrophic components of energy are negligible, except for the isallobaric kinetic energy of motion of the vortex as a whole.

An increase in the mutual energy of such a system can take place only at the expense of the isallobaric energy, *i.e.*, one or more of the vortices must slow down.

The simplest case is that of the interaction of two vortices, but the theory extends to more complex systems, for the total mutual energy of a vortex system is the sum of the mutual energies of the constituent vortices taken in pairs.

13. The two-vortex problem

On the hypothesis that the sum of the isallobaric and mutual energies of two vortices is constant we may write the equation of energy for a two-vortex system

$$\pm 8\sqrt{K_1 K_2}\,\frac{R_1^2 R_2^2}{(R_1^2+R_2^2)^2}\left[1-x^2/2\,(R_1^2+R_2^2)\right]e^{-x^2/2\,(R_1^2+R_2^2)} + \frac{K_1 C_1^2}{\lambda^2 R_1^2}+\frac{K_2 C_2^2}{\lambda^2 R_2^2}=\text{Const} \quad . \quad . \quad (32)$$

where C_1 and C_2 are the speeds of vortex 1 and 2 respectively, and x is their separation.

This energy equation is exactly analogous to the similar equation in the dynamics of attracting bodies which equates the sum of the kinetic energies, plus the potential (mutual) energy, to a constant. The general two-vortex problem, analogous to the two-body problem in dynamics, of determining the subsequent motion of two vortices, given their position and motion at one instant, is complex in the most general possible case.

If the vortices are rotating about some point with constant separation, such motion could persist without violating the energy equation, as the mutual energy is a function of the separation only.

Consider the simplest case of vortex 2 stationary, and vortex 1 moving in a line of centres with speed $C=\frac{dx}{dt}$. The speed at infinity is C_∞. Then the equation of energy may be written

$$\frac{K_1 C_\infty^2}{\lambda^2 R_1^2}=\frac{K_1 C^2}{\lambda^2 R_1^2}\pm 8\sqrt{K_1 K_2}\,\frac{R_1^2 R_2^2}{(R_1^2+R_2^2)^2}\left[1-x^2/2\,(R_1^2+R_2^2)\right]e^{-x^2/2(R_1^2+R_2^2)}$$

In the case of like vortices, there is an initial acceleration up to a separation $x=2\sqrt{(R_1^2+R_2^2)}$ where the speed achieves its maximum value

$$C^2=C_\infty^2+\frac{8\lambda^2 R_1^4 R_2^2}{(R_1^2+R_2^2)^2}\sqrt{K_2/K_1}\,.\,e^{-2}$$

For closer approach the vortex slows up, and reaches the speed C_∞ at $x=\sqrt{2\,(R_1^2+R_2^2)}$. The speed will become zero for a separation given by

$$C_\infty^2=\frac{8\lambda^2 R_1^4 R_2^2}{(R_1^2+R_2^2)^2}\,.\sqrt{K_2/K_1}\,.\left[1-x^2/2\,(R_1^2 R_2^2)\right]e^{-x^2/2\,(R_1^2+R_2^2)} \quad . \quad . \quad (33)$$

This point will coincide with the centre of the stationary vortex if

$$C_\infty^2 = \frac{8\lambda^2 R_1^4 R_2^2}{(R_1^2 + R_2^2)^2} \cdot \sqrt{K_2/K_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

If the initial speed is less than that given by (34) the vortex will come to rest at a separation given by (33). The equilibrium will be unstable, however, for any slight displacement away from the centre of the stationary vortex will lead to reduced mutual energy and hence increased isallobaric energy, so that it will continue to move away from the stationary centre.

In the case where the vortices are of opposite sign, the moving vortex will decelerate initially, and the minimum speed will be

$$C^2 = C_\infty^2 - \frac{8\lambda^2 R_1^4 R_2^2}{(R_1^2 + R_2^2)^2} \sqrt{K_2/K_1}\, e^{-2} \quad . \quad . \quad . \quad (35)$$

Unless

$$C_\infty^2 \geq \frac{8\lambda^2 R_1^4 R_2^2}{(R_1^2 + R_2^2)^2} \sqrt{K_2/K_1}\, e^{-2}$$

the moving vortex will not have sufficient isallobaric energy to surmount the potential barrier. It will be observed from (35) that the initial speed required is small in the case of a vortex of small spread and high inner energy moving towards one of large spread and low inner energy. As an example, a cyclone of spread 250 km in latitude 60° would require an initial speed of 6 m/sec to surmount the barrier of an anticyclone of spread 1,250 km and the same intensity. Once the potential barrier is surmounted the speed is accelerated, and the moving vortex attains a speed

$$C^2 = C_\infty^2 + \frac{8\lambda^2 R_1^4 R_2^2}{(R_1^2 + R_2^2)^2} \sqrt{K_2/K_1}$$

at the centre of the stationary vortex. In the above example the speed at the centre would be 17 m/sec. The vortices would then separate with decreasing speed until a speed zero is achieved at $r = 2\sqrt{(R_1^2 + R_2^2)}$. This is a point of unstable equilibrium, and the cyclone may continue to separate with accelerating speed until it achieves its initial velocity at infinity of 6 m/sec or be attracted back to the centre of the stationary vortex.

Capture of a cyclone by an anticyclone is possible if the potential barrier is increasing with time, as the moving vortex cannot acquire enough isallobaric energy during the initial stages when attraction is weak to surmount the barrier when it becomes strong. In this case the cyclone will oscillate back and forth through the centre of the anticyclone.

No such similar capture is possible in the case of vortices of like sign.

We must not conclude from this that like vortices in the free atmosphere *never* merge. It may be possible for two cyclones to amalgamate, the mutual energy of the system becoming incorporated in the inner energy of the new vortex.

However, capture is not possible in which the two vortices preserve their individual identities.

14. The interaction of tilted vortices

We have dealt hitherto with vortices whose axes are vertical. A little reflection will show that where the axis of one vortex is tilted relative to that of the other, the mutual kinetic energy will, broadly speaking, be less, and the forces of attraction or repulsion less in consequence. Suppose at the surface the centres of two like vortices are close together. The mutual kinetic energy of the surface layers will be positive. But with tilted axes the centres at height may be more widely separated, and here the mutual energy may be negative, or, at least, not so strongly positive as for the surface layers.

Hence the effect of relative tilt in the axes of two vortices is to reduce the potential barrier and to flatten out the mutual energy curve (*i.e.*, to increase the standard deviation).

15. The power vortex

The discussion has hitherto been based entirely on an assumed normal pressure profile. The normal profile calls for wind speeds decreasing outwards from the ring of maximum wind rather more rapidly than is observed in atmospheric vortices. It is of interest to study the shape of the mutual energy curve for vortices more closely approximating the actual profile of extratropical disturbances.

A suitable model, the power vortex, has been briefly discussed. The mutual energy curve for two identical power vortices, index 2, is shown in Fig. 3. Distances are expressed in units of one spread radius.

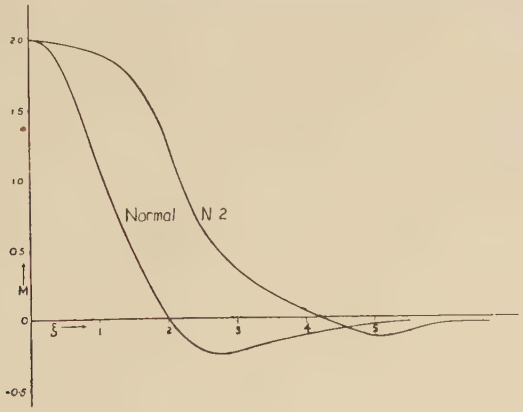

Figure 3. Mutual energy for normal and power vortices.

The general shape of the curve is the same as for normal vortices, but the height to the potential barrier is lower, and the barrier occurs at a greater separation, 5 spread radii, compared with 2·83 spread radii for the normal vortex. The use

of the normal vortex as an atmospheric model thus tends to overstate the magnitude of the potential barrier, and to understate the separation distance, but the general qualitative features of the mutual interaction are correctly given.

16. Acknowledgments

The writer is indebted to Mr. C. H. B. Priestley for discussions leading to clarification of the points raised in this paper. The work is part of the research programme of the Section of Meteorological Physics, Commonwealth Scientific and Industrial Research Organization.

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Variations in thunderstorm severity in Great Britain

By J. S. FORREST

(Manuscript received 14 October 1949)

Summary

The paper gives an account of variations in thunderstorm severity in Great Britain based on statistics of breakdowns caused by lightning on the Grid system during the 14-year period 1934 - 1947. Data are given for the seasonal and diurnal variations and for the fluctuations from year to year. The breakdown data are compared with the corresponding statistics of thunderstorm-days, and it is shown that the thunderstorm-day observations, if used directly, do not give an adequate indication of storm severity. It is proposed to introduce a " severity index " (s) such that the severity is proportional to T^s, where T is the number of thunderstorm-days. For the data examined, s has values of 1·4 and 1·9.

1. Introduction

Lightning is a common cause of breakdowns on overhead electric-power systems, and records of such breakdowns can be used to provide information on the frequency of occurrence and the severity * of thunderstorms. It should be noted that breakdowns are caused only by lightning flashes to earth. Further, only a certain small fraction of the flashes to earth causes breakdowns ; this is because all flashes to earth are not intercepted by power lines, and in addition, lightning may strike a line, especially a high-voltage line, without causing a breakdown in the usual sense (Forrest 1950). In spite of these limitations, however, it is considered that overhead-line breakdown records probably provide the best available measure of thunderstorm severity.

The statistics of the British Grid system are particularly suitable for this purpose as the Grid lines are fairly uniformly distributed over the whole country (except the North of Scotland), and furthermore, the same general form of construction is used throughout. Records are now available of 1,157 breakdowns which were caused by lightning on the 5,900 miles of line of the Grid system during the 14-year period 1934 - 1947. It is of interest, therefore, to present these statistics as measures of the fluctuations of thunderstorm severity from year to year and of the seasonal and diurnal variations.

Records of " thunderstorm-days " are often the only available guide to thunderstorm severity and are often used to estimate the probability of transmission line breakdowns due to lightning (Harder and Clayton 1949). Accordingly, the observations of thunderstorm-days at selected meteorological stations for the same 14-year period have been extracted for comparison with the power-system data.

* The term " severity " is used in this paper to mean the number of breakdowns or, in more general terms, the amount of damage due to lightning ; it is presumably directly proportional to the number of lightning flashes to earth.

2. Basic data

(a) *Thunderstorm-days*

The eight stations—Renfrew, Eskdalemuir, Tynemouth, Birmingham, Kew, Southampton, Stonyhurst and Long Ashton were selected as representing the areas covered by the Grid system, taking into account, as far as possible, the density of the overhead-line network in various parts of the country. The location of the stations in relation to the overhead-line network was considered to be the most important factor in their selection. Some of these stations do not maintain a continuous 24-hour watch, but this is not a serious disadvantage, as we are mainly concerned with *variations* in the number of thunderstorm-days from year to year and from summer to winter, and not with the actual numbers. The thunderstorm-days at each station for each month of the 14 years were obtained from the *Monthly Weather Report* of the Meteorological Office, and the sum of the thunderstorm-days for the eight stations was taken as a measure of the thunderstorm frequency for the whole area covered by the Grid. Table I gives the data obtained in this way.

TABLE I. Total thunderstorm-days at eight stations

Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
1934	0	0	2	11	3	13	28	16	17	4	0	1	95
1935	1	3	0	12	1	34	6	9	18	10	3	4	101
1936	0	0	2	1	14	31	30	7	20	3	3	3	114
1937	2	4	8	6	20	13	13	18	11	4	0	1	100
1938	5	2	1	0	6	11	31	36	4	6	7	2	111
1939	1	2	4	4	7	13	39	31	8	8	4	2	123
1940	0	1	0	14	20	10	32	0	4	3	5	3	92
1941	1	6	11	7	22	18	22	17	2	1	1	2	110
1942	2	0	5	9	13	5	8	19	7	9	0	0	77
1943	3	3	0	3	11	16	21	17	13	6	3	2	98
1944	2	1	0	5	13	13	14	16	8	8	1	2	83
1945	2	6	0	1	25	23	17	9	11	4	2	2	102
1946	2	8	1	6	18	32	14	19	11	6	3	0	120
1947	2	0	1	3	35	20	28	2	2	0	0	3	96
Total	23	36	35	82	208	252	303	216	136	72	32	27	1,422

(b) *Power system breakdowns*

The number of breakdowns on the Grid system due to lightning for each month of the 14 years is given in Table II. All the breakdowns recorded in this table are known, with quite a high degree of certainty, from their characteristics and from other evidence (Forrest 1943), to be caused by lightning. Further, the time of occurrence and the location of the breakdown are known.

TABLE II. Power-system breakdowns due to lightning

Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total	Corrected Total
1934	1	0	2	5	4	4	25	10	18	2	0	0	71	89
1935	2	2	0	5	2	37	4	8	14	3	0	2	79	98
1936	0	0	0	0	8	61	22	3	15	8	1	1	119	146
1937	0	0	2	1	3	3	6	23	1	1	0	0	40	49
1938	1	0	0	0	3	0	19	55	1	1	1	0	81	95
1939	0	2	1	1	2	1	65	30	5	7	1	1	116	135
1940	0	0	0	6	12	7	25	0	0	2	1	0	53	60
1941	0	0	0	1	1	24	47	9	2	6	0	0	90	96
1942	0	0	2	6	4	6	5	16	4	3	0	0	46	48
1943	2	0	1	0	4	12	29	7	19	1	3	1	79	80
1944	0	0	0	3	28	9	12	6	4	4	2	0	68	69
1945	13	2	0	3	13	11	32	17	9	4	2	2	108	109
1946	1	4	0	11	10	26	20	16	5	2	1	2	98	99
1947	1	0	0	0	35	23	42	2	3	0	0	3	109	109
Total	21	10	8	42	129	224	353	202	100	44	12	12	1,157	1,282

In the following sections a detailed comparison will be made of the data given in Tables I and II.

3. Variation from year to year

Although there was no major change in the Grid system in the 14-year period under review, the length of line increased to some extent, and it is desirable when considering long-period variations to adjust the breakdown figures to a common line length on the assumption that the number of breakdowns is directly proportional to the length of line in service. The annual number of breakdowns (corrected to the 1947 mileage) is then as given in the final column of Table II. These figures are compared graphically in Fig. 1 with the corresponding total thunderstorm-days for the eight stations. It will be seen that both curves follow the same general trend. Thus the high thunderstorm-day frequencies in 1936 and 1939 correspond to the greatest numbers of breakdowns in the 14-year period. In 1942, when the number of storm-days was the lowest in the period under review, the lowest number of breakdowns was also recorded. In 1946, on the other hand, the agreement is not so good, as although thunderstorm-days were frequent, the number of breakdowns was relatively small. The most noteworthy feature, however, is that the breakdowns vary over a much greater range than the thunderstorm-days as shown below :—

	Range	*Mean*	*Standard Deviation*
Thunderstorm-days ..	77-123	101·6	12·6
Breakdowns	48-146	91·6	28

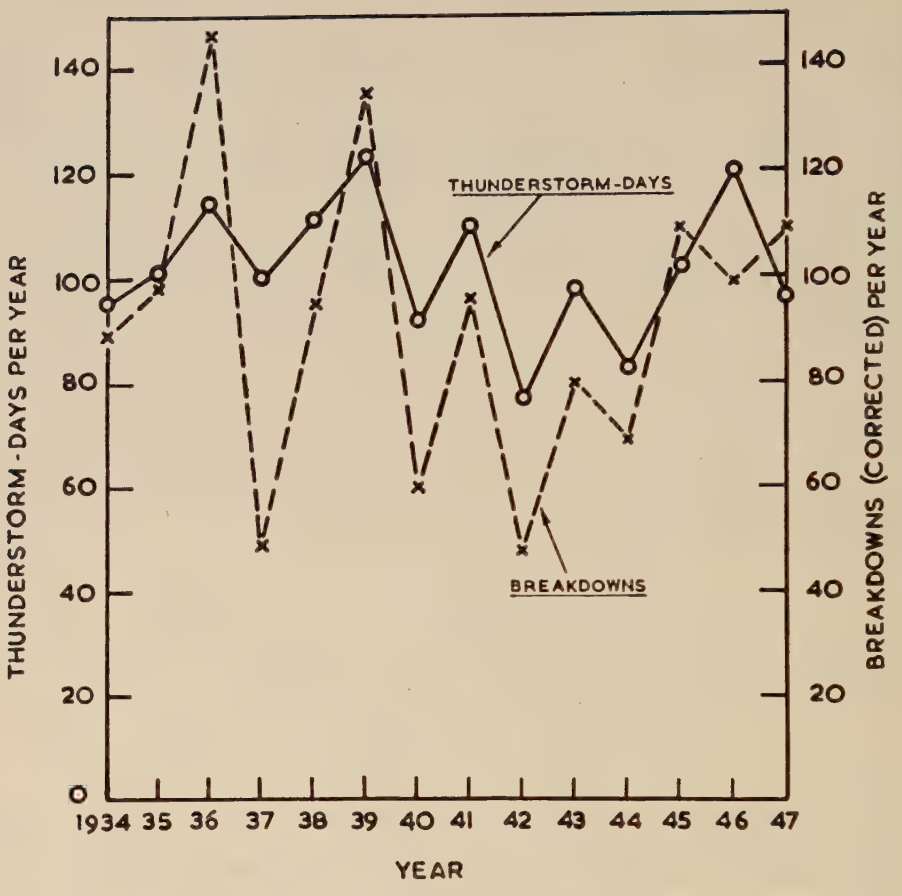

Figure 1. Comparison between annual number of breakdowns and annual number of thunderstorm-days.

The correlation coefficient between thunderstorm-days and breakdowns is 0·74 ; a highly significant value for 14 pairs of observations. It may be remarked in passing that the correction of the breakdown figures for line length increases the correlation with thunderstorm-days, the correlation using the unweighted data being 0·69.

4. Seasonal variation

The total number of thunderstorm-days in each month of the year for the eight stations and for the 14-year period is compared in Fig. 2 with the corresponding monthly totals of breakdowns. It is seen that the agreement between the two sets of data is very good ; both exhibit a pronounced maximum in July and a minimum in the winter months. There is a slight anomaly in January, however, when the thunderstorm-days are fewest and the breakdown frequency is relatively high. This does not appear to be significant, however, and is accounted for by one exceptional storm, in 1945, which caused 13 breakdowns — 10 times

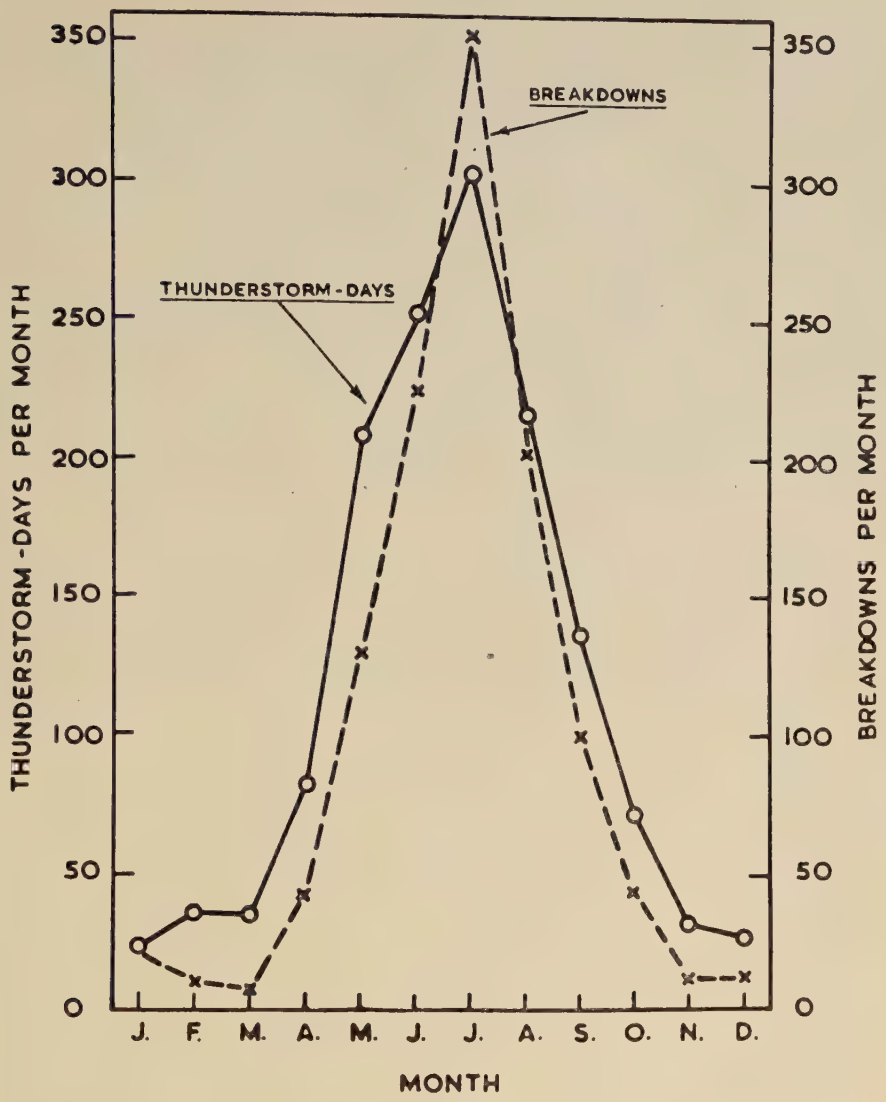

Figure 2. Total thunderstorm-days and total breakdowns per month ; 1934-1947.

the January average. The most significant difference between the two curves is, as in the case of the annual data, that the range of variation is greater for breakdowns than for thunderstorm-days. Thus the thunderstorm-days vary from 23 (in January) to 303 (in July) — a ratio of 13 to 1 ; and the breakdowns vary from 353 (in July) to 8 (in March) — a ratio of 44 to 1. Nevertheless, the correlation between thunderstorm-days and breakdowns per month is high, the value of the correlation coefficient being 0·97 — higher than in the case of the annual figures.

A detailed examination of Tables I and II shows clearly how the varying

thunderstorm frequency in individual months is reflected in the breakdown data. For example, 1935 was noteworthy in that severe thunderstorms occurred in June ; the number of thunderstorm-days was 1·9 times the average and the number of breakdowns 2·3 times the average. The maximum number of thunderstorm-days (39 for the eight stations) in any month occurred in July 1939 — in this month the maximum number of breakdowns (65) was also recorded. Further, August 1940 is exceptional as the only August in the 14 years with no days with thunder ; it was also the only August with no breakdowns.

The figures for May and August are interesting ; May had 208 days with thunder — nearly as many as August which had 216. On the other hand, there were only 129 breakdowns in May compared to the August figure of 202. It is well known that, in May, instability is common and thunderstorms correspondingly frequent, but as temperatures are normally not yet high the storms are neither prolonged nor severe. The above figures therefore support the contention that thunderstorm-day statistics do not do justice to the severity of the storm.

5. Diurnal variation

Table III shows the time of occurrence of 651 breakdowns due to lightning in all parts of the Grid system during the eight years 1940-1947. (A period of eight years was found to be adequate when considering diurnal variation.) The summer (April-September) and winter (October-March) breakdowns are given separately and the diurnal variation of the totals is shown in Fig. 3. A pronounced

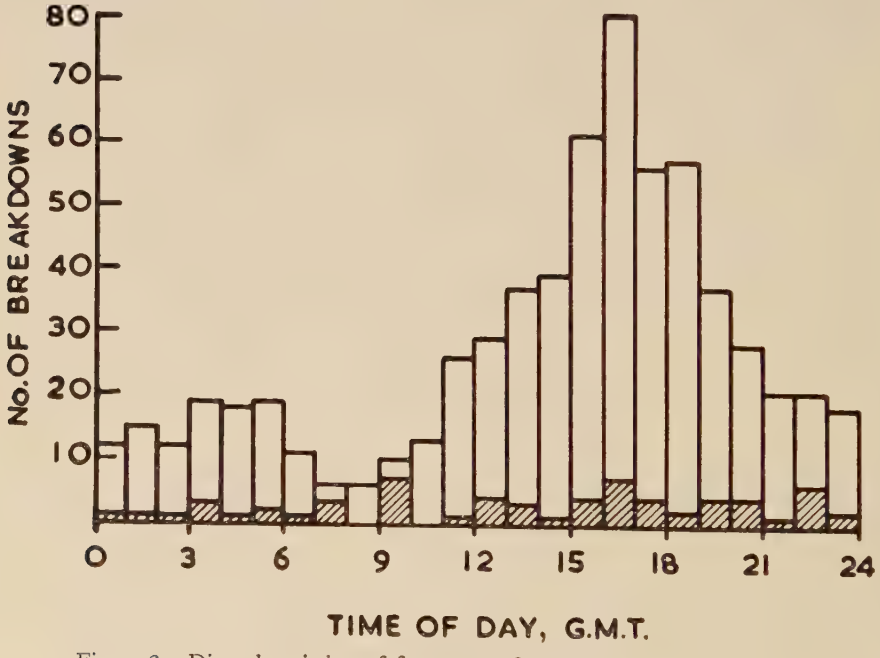

Figure 3. Diurnal variation of frequency of occurrence of breakdowns ; 1940-1947. The hatched areas show the winter breakdowns.

maximum occurs in the afternoon between 1600 and 1700 G.M.T., while the minimum is between 0700 and 0900 G.M.T. It is interesting to note that there is a secondary maximum in the early hours of the morning, between 0300 and 0600. The winter breakdowns are distributed more uniformly throughout the 24 hours and do not exhibit a maximum in the afternoon.

Meteorological data on the diurnal variation in the frequency of thunder are not available to enable a detailed comparison to be made with the breakdown data, but the information which has been published is in general agreement with the results shown in Fig. 3. For example, Bishop (1947) gives a histogram showing the number of occasions on which thunder was heard at Kew in each two-hourly interval of the day for the period 1910-1935. This shows the pronounced afternoon maximum (apparently an hour or so earlier than that of Fig. 3), and the early morning minimum (from 0400 to 1000). There is no indication of a secondary maximum in the early hours of the morning, but possibly the breakdown data are more reliable at this time. A significant feature, however, is that the ratio of the maximum to the minimum frequency is approximately the same for the occasions on which thunder was heard as for the breakdowns — namely 13 : 1. This is to be expected, as we are not now dealing with thunderstorm-*days*.

TABLE III. Diurnal variation of frequency of breakdowns
S — Summer (April-Sept.)
W — Winter (Jan.-March, Oct.-Dec.)

Time	1940		1941		1942		1943		1944		1945		1946		1947		Total (8 yrs.)	
G.M.T.	S	W	S	W	S	W	S	W	S	W	S	W	S	W	S	W	S	W
0-1	1	0	1	0	2	0	2	0	1	0	2	1	1	0	1	0	11	1
1-2	0	0	0	0	0	0	2	0	2	0	4	1	1	0	5	0	14	1
2-3	1	0	2	0	0	0	5	0	0	0	0	1	0	0	3	0	11	1
3-4	0	0	2	0	3	0	1	0	2	0	5	2	0	1	3	0	16	3
4-5	0	0	3	0	1	0	2	0	5	0	2	0	1	1	3	0	17	1
5-6	0	0	2	0	0	0	0	1	1	0	1	1	4	0	9	0	17	2
6-7	1	0	1	0	0	0	0	0	1	0	2	0	1	1	4	0	10	1
7-8	0	0	1	0	0	1	0	2	0	0	1	0	0	0	1	0	3	3
8-9	0	0	0	0	0	0	0	0	1	0	2	0	0	0	3	0	6	0
9-10	0	1	0	3	0	0	0	0	0	1	1	0	1	1	1	1	3	7
10-11	1	0	0	0	1	0	2	0	0	0	0	0	2	0	7	0	13	0
11-12	5	0	2	0	1	0	3	0	1	0	3	1	3	0	7	0	25	1
12-13	8	1	2	0	0	0	1	0	2	0	1	1	7	2	4	0	25	4
13-14	6	0	5	1	2	0	3	1	4	0	1	1	8	0	5	0	34	3
14-15	0	0	8	0	5	0	4	0	5	1	4	0	5	0	7	0	38	1
15-16	9	0	8	0	6	1	2	0	8	1	5	1	10	1	9	0	57	4
16-17	10	0	16	0	3	1	6	0	5	2	9	3	15	0	9	1	73	7
17-18	3	0	10	1	8	1	13	0	5	0	3	1	3	0	7	1	52	4
18-19	2	0	5	0	4	0	5	1	9	0	16	0	9	1	5	0	55	2
19-20	1	0	8	1	2	1	6	0	3	0	6	1	6	1	1	0	33	4
20-21	1	0	1	0	2	0	3	1	6	1	4	1	2	1	5	0	24	4
21-22	1	0	2	0	1	0	4	1	1	0	8	0	1	0	2	0	20	1
22-23	0	1	3	0	0	0	1	0	0	0	4	4	4	0	3	1	15	6
23-24	0	0	2	0	0	0	6	1	0	0	3	1	4	0	1	0	16	2
Total	50	3	84	6	41	5	71	8	62	6	87	21	88	10	105	4	588	63

6. Relationship between breakdowns and thunderstorm-days

We saw in Sections 3 and 4 that the most significant difference between the two sets of data was that the thunderstorm-days varied over a smaller range than the breakdowns. This could result from a power law relationship,

$$N = kT^s$$

where N is the number of breakdowns,
T the corresponding number of thunderstorm-days,
k a constant depending on power system conditions, *etc.*
and s the "severity index" (>1).

The annual totals have been plotted on log paper in Fig. 4 and although the points show considerable scatter, it is clear that the observations cannot be satisfactorily represented by assuming direct proportionality between thunderstorm-days

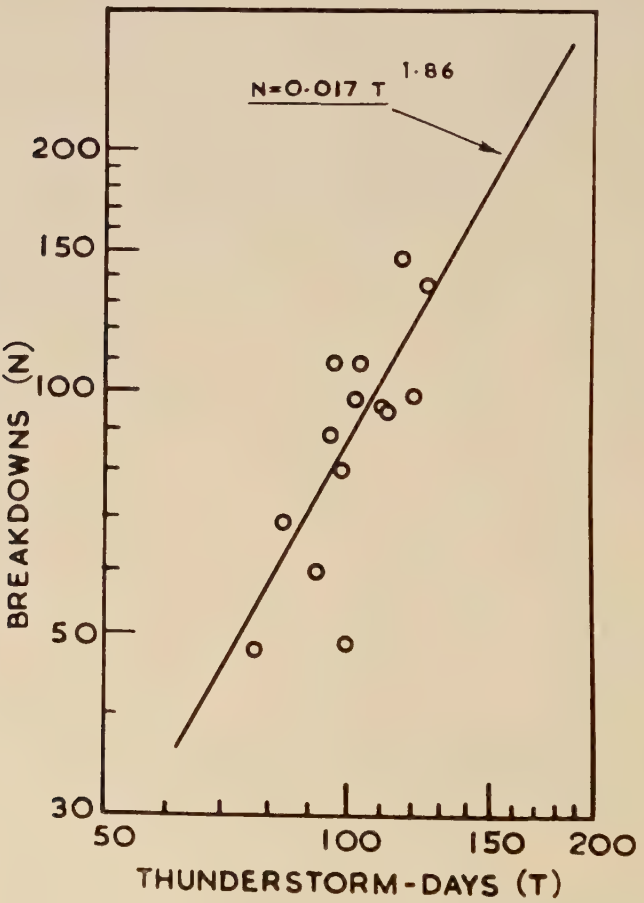

Figure 4. Relation between annual numbers of thunderstorm-days and annual numbers of breakdowns.

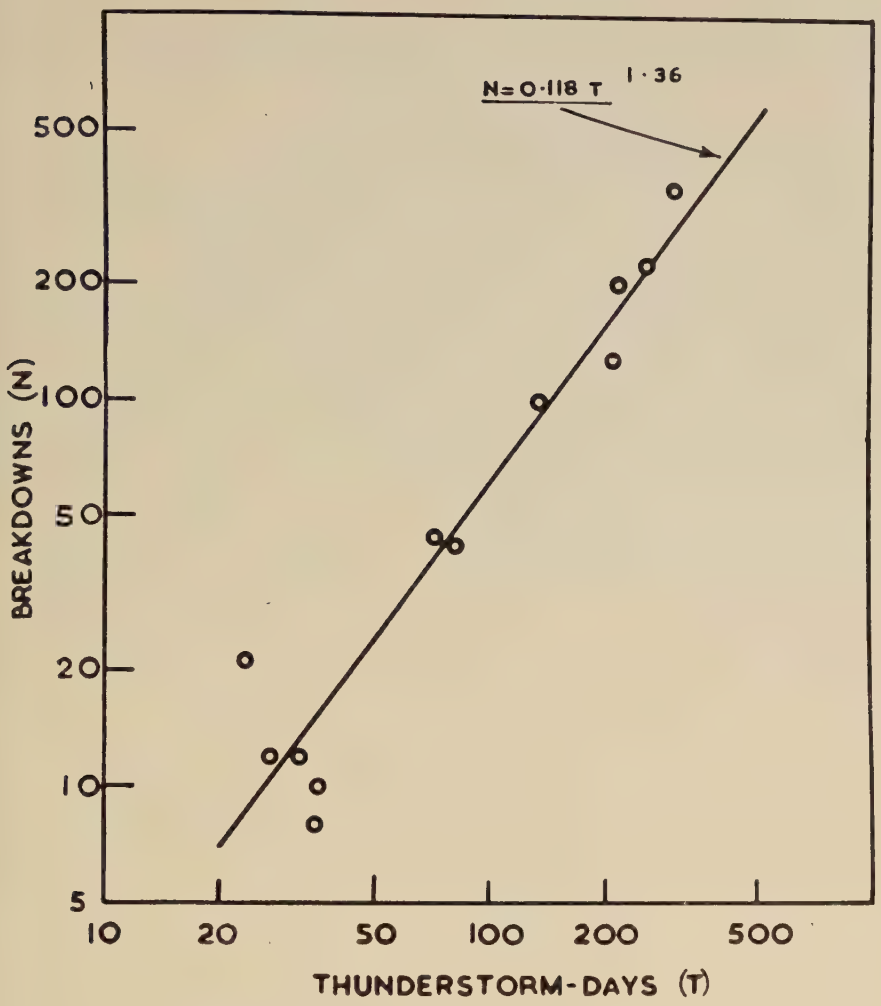

Figure 5. Relation between monthly totals of thunderstorm-days and of breakdowns.

and breakdowns. The " best straight line " drawn by the method of least squares is shown in Fig. 4 and has the equation

$$N = 0{\cdot}017T^{1{\cdot}86},$$

that is, the severity index is approximately 1·9.

The monthly totals have been treated in the same way in Fig. 5. In this case, as we saw in Section 4, the correlation is high (0·97) and the observations are well-represented by the straight line shown in the diagram. The greatest deviations from the straight line are due to the figures for January and May which, as we have

already noted, are somewhat exceptional. The equation of the line, using the method of least squares and taking the " exceptional " observations into account, is

$$N = 0{\cdot}118T^{1{\cdot}36};$$

thus the severity index is approximately 1·4.

It should be noted that it is reasonable for the severity index deduced from the annual totals to have a different value from that obtained from the monthly totals. In the former case, we are considering the difference between severe and mild years, while in the latter case it is a question of the different characteristics of summer and winter thunderstorms. The physical interpretation of the severity index is that when thunderstorm-days increase in frequency the actual numbers of storms per day, and/or the duration of the storms, also increase. Thus, considering the monthly data, if we assume that the severity (*i.e.*, the number of breakdowns) is directly proportional to the number of storms, then it follows that the number of storms per thunderstorm-day is proportional to $T^{0{\cdot}4}$. For example, it is seen from Table I that the ratio of the number of thunderstorm-days in July to the number in January is 13 ; hence in July the number of storms per thunderstorm-day (or the duration of the storms) is $13^{0{\cdot}4}$, that is, 2·8 times that in January. In more general terms, there are 10 times as many days with thunder in the summer months as in the winter months, so that the duration of summer storms is on the average $10^{0{\cdot}4}$, that is, $2\frac{1}{2}$ times that of winter storms. This deduction seems to be in reasonable agreement with experience.

7. Conclusions

Statistics of breakdowns due to lightning on extensive overhead power systems provide a useful measure of fluctuations in thunderstorm severity from year to year and of seasonal and diurnal variations.

The number of breakdowns (which is taken as a measure of storm severity) is found to be proportional to T^s, where T is the corresponding number of thunderstorm-days, and s (the " severity index ") has values of 1·4 and 1·9 for the data examined. Hence, in any case for which thunderstorm *severity* is important — such as danger to life, damage to electrical installations, aircraft or buildings, and the assessment of fire and other lightning risks for insurance purposes — statistics of thunderstorm-days may be misleading if used directly. A better estimate of the danger or damage likely to be incurred can however be obtained by the use of an appropriate severity index greater than unity.

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Some further considerations of the measurement and indirect evaluation of natural evaporation

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SUMMARY

A previous observational study (Pasquill 1949 a) has been supplemented by further measurements of evaporation and the associated vertical profiles of wind velocity and vapour pressure in the first two metres above a clayland pasture with grass of moderate length, mainly in effectively adiabatic conditions of flow. The evaporation measurements, using a simple form of soil evaporimeter, included two series of statistically designed measurements, in order further to examine the "reliability" of the technique. The data so obtained, though not comprehensive, indicate no substantial error due to the necessary isolation of test soil from underlying layers, but there is a suggestion that the rates of evaporation ultimately adopted might be underestimates, owing to unavoidable disturbance of root and soil continuity in the vertical at the perimeters of the soil cores contained in the evaporimeters.

The observed rates of evaporation (E_O) are compared with those computed (E_C) from the following formula, which is based on the Prandtl form of the law relating drag and wind profile over a rough surface in adiabatic conditions,

$$E_C = \frac{\rho k^2 (u_2 - u_1)(q_1 - q_2)}{\left[\log\left(\frac{z_2 - d}{z_1 - d}\right)\right]^2}$$

where u_1, u_2, q_1, q_2 are the wind velocities and specific humidities at heights z_1, z_2 above the soil surface, d is a 'zero point displacement' associated with the vegetation cover, ρ is air density and k is von Kármán's constant, of magnitude 0·4. For the present measurements the usual "roughness parameter," deduced from the wind profiles, was 0·51 cm as compared with 0·25 cm for the previous series. The two series include 19 separate determinations of rate of evaporation over periods of about one hour, and in terms of different sections of the observed profiles, provide 41 values of E_C/E_O in effectively adiabatic conditions, with wind velocities at a height of 2 m ranging from 3 to 9 m/sec. The mean value of E_C/E_O is 0·99 and the individual deviations from 1·0 are within 10, 20 and 30 per cent in 24, 66 and 90 per cent of cases. It is demonstrated that deviations of this order are to be expected from local variations in the various factors involved.

Although some *a priori* uncertainty remains concerning the precise validity of the evaporation measurements, the consistency so demonstrated in terms of a semi-theoretical treatment which has been established in other connections (Calder 1949) suggests that the ultimate result may be accepted with confidence. Brief consideration is then given to the validity and practicability of applying the above formula to evaporation from areas of growing crops.

1. INTRODUCTION

An observational study during the spring of 1948 (Pasquill 1949a) provided data on some of the features involved in the turbulent transport of water vapour over a short grass surface and on the validity of formulae prescribing natural evaporation in terms of meteorological factors. The investigation was resumed in the spring of 1949 with the aims, in particular, of justifying more firmly the method adopted for the evaporation measurements, and of carrying out these and other relevant measurements for a range of grass lengths. Owing primarily to the generally dry state of ground then existing it was found practicable to apply the

technique of evaporation measurement on only two occasions ; during these occasions the grass was only slightly longer than during the initial investigation, and, moreover, no substantial departures from adiabatic conditions were encountered.

Thus, the present investigation does not directly subscribe to the important problem of generally extending the earlier results to a surface roughness approaching the magnitude associated with crops in full growth. It does, however, add somewhat to the scanty data as yet available on direct measurement of natural evaporation and substantially reinforces the evidence for the validity of the formula for evaporation based on the Prandtl form of the law relating wind profile and surface drag in adiabatic conditions. Furthermore, some observations of the spatial variations of wind speed and vapour pressure permit some appreciation of the reliability of individual observations. Finally, despite the lack of relevant observational data, a brief discussion is given concerning the indirect evaluation of evaporation from areas of growing crops.

2. Observational aspects

Observations of the vertical profiles of wind velocity, air temperature and vapour pressure, and of water loss from soil evaporimeters of simple design, were carried out on the site employed previously, again avoiding wind directions in the obstructed N.-E. quarter. The profile observations employed anemometers and distant-reading thermocouple psychrometers identical with those described previously (Pasquill 1949a, b) and the technique of observation differed only slightly. Differences in dry- and wet-bulb temperatures over the height intervals 200-100, 150-37½ and 50 - 25 cm were read directly (nominally to 0·01°F), while absolute values of wet-bulb temperature were measured at the higher level in each interval and dry-bulb temperature at 200 cm (nominally to 0·1°F). The nominal temperature readings were subsequently corrected on the basis of regular calibrations of the thermocouple equipment against standard mercury-in-glass thermometers, approximate corrections also being applied, as before, for the thermal lag of the temperature elements.

One reading of each temperature or temperature difference was taken every two minutes, in systematic order, for a period of 20 min during which the anemometers (exposed at six heights as above) were in continuous operation. Anemometers and psychrometers were then interchanged in each of the above height intervals, so as to eliminate defects in instrumental performance, and the process repeated after allowing a period of 10 min for adjustment of the psychrometers to their new environments. The means of the 20 individual temperature readings and of the two anemometer runs were taken as representative of the whole period of observation (approximately one hour). A thorough wind-tunnel calibration of the anemometers was applied retrospectively.

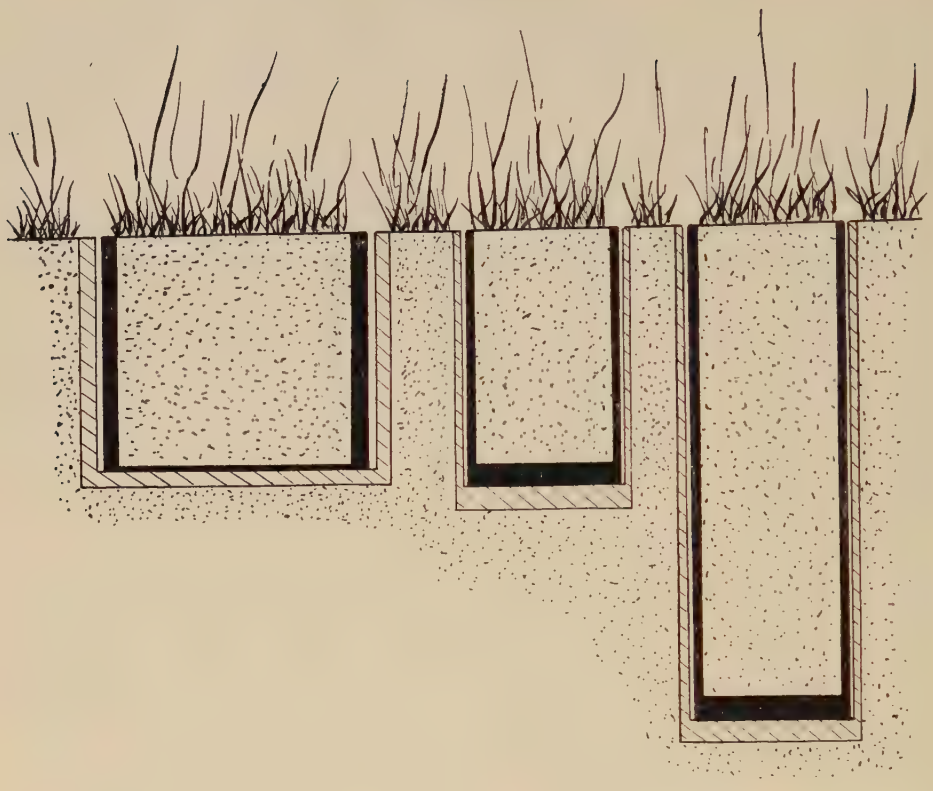
The evaporation measurements were identical in principle with those described previously, but with some modifications in design, and a fairly detailed description of this aspect seems desirable. The general principle, it may be recalled, is that

on a clay soil such as that which exists on the present site, and which characteristically dries out progressively from the surface downward, it may be possible to select occasions when isolation of a shallow block of soil from surrounding and underlying soil, while maintaining the original exposure at the surface, will not for some time influence the rate of loss of water from the test soil. Clearly one essential condition is that the progressive drying shall not have extended below the depth at which the soil is isolated.

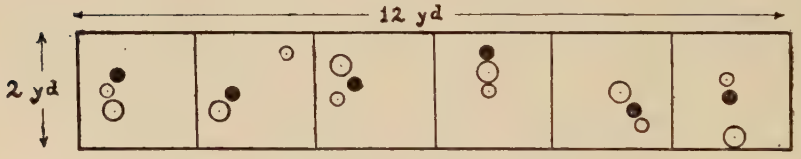
The application of the technique presents two main problems. In the first place there are the mechanical features of the size, extraction, exposure and weighing of the soil. In the second place there is the designing of " control technique " on which to justify the adoption of the measured water losses as uninfluenced by the necessary isolation of the test soil. The mechanical features were similar to those of the initial investigation, soil cores of circular cross-section being extracted with " undisturbed core " samplers of conventional design, suitably exposed in Tufnol containers and weighed at intervals on an indicating balance. Further details are shown in Fig. 1 (a).

For the measurements over periods of one hour, which are subsequently related to the profile observations, the evaporimeters were, as before, nominally 4 in. in diameter and depth. The cutting diameter of the soil sampler was actually $3\frac{15}{16}$ in., and this was adopted as the effective diameter in computing rates of evaporation from the observed water losses. Two types of control measurement were suggested and applied to some degree in the initial investigation. The first method considers the effect of a change in the *depth* of the soil cores, the second the effect of a difference in *age* of the cores, the idea being that the effects of the isolation of the cores should show up with decreasing depth and increasing age. In the present measurements an attempt was made to apply the " depth " test in a more comprehensive fashion than hitherto, by using evaporimeters of 8 in. depth, with necessarily smaller diameter (nominally $2\frac{3}{8}$ in., cutting diameter of sampler $2\frac{5}{16}$ in.) so that the total weights were still within the capacity of the indicating balance, and 4 in. evaporimeters of the same cross-section for direct comparison. The final combination thus provided in addition a comparison of evaporimeters of the same depth but different cross sectional areas. Owing to the smaller evaporating areas presented by the additional evaporimeters the control measurements were made over periods of several hours or more.

In practice six evaporimeters of each type were prepared and exposed concurrently and in view of previous indications of considerable point-to-point variations in local rate of evaporation it was decided to set out the evaporimeters according to some suitable statistical design. The procedure adopted was to mark out a rectangle of 12 x 2 yd, 10 - 20 yd from the position of the profile apparatus in the anticipated upwind direction, with the long side lying across wind. This rectangle was divided into six plots each 2 x 2 yd, in each of which was set out one evaporimeter of each type, the plots and corresponding evaporimeters being numbered 1 - 6. In each plot the three evaporimeters were arranged as close together as possible in a random order. Soil cores were employed only if inspection showed



(a) Soil Evaporimeters.
Tufnol container holding soil core.
\\\\ Tufnol lining embedded in ground.



(b) Approximate disposition of evaporimeters (22.3.49)
● indicates evaporimeter of 8 in. depth.

Figure 1.

them to be completely intact, apart from shallow cracks on the outside, which appeared to be unavoidable. Thus in some cases extractions had to be repeated and a wider separation in the group had to be accepted.

Satisfactory extraction of the soil cores was generally practicable up to mid-April, during which period two series of measurements were carried out. It may

be noted that on both occasions the root growth frequently extended below a depth of 4 in., but only occasionally below 8 in. The approximate disposition of the evaporimeters for one of these is indicated in Fig. 1 (b). Subsequent attempts were unsuccessful, owing to the increasing dryness of the surface layers of soil and the consequent breaking up of soil cores on extraction. Indeed the experience gained during the present season demonstrated most strikingly the limited extent of the drying period during which soil cores can be extracted satisfactorily.

3. Reliability of the evaporation measurements

The individual evaporation losses were usually of the order of 1 gm for the hourly measurements, and frequently more than this for the longer period measurements, even for the evaporimeters of small cross-section. These losses were read to 0·01 gm on an indicating balance the scale of which was calibrated against reliable weights. A few test readings indicated that errors in individual readings were less than 0·02 gm. The accuracy of reading was thus quite adequate for an investigation of the present type.

We may now consider the implications of the control measurements as regards the ultimate reliability of the results obtained. It will be seen that some uncertainty remains. The two sets of data are summarised in Table I below, the water losses being expressed in the form of an equivalent depth, so as to permit direct comparison of the evaporimeters of different cross-sectional areas.

TABLE I. Water losses in mm from various evaporimeters

Date of setting out evaporimeters	Period of Measurement	Evaporimeter		Plot and Evaporimeter Number						
		Depth inches	Diam. inches	1	2	3	4	5	6	Mean
22/3/49	1610 G.M.T. 22/3/49 to 1545 G.M.T. 24/3/49	4	4	2·67	2·43	3·08	2·70	1·94	2·65	2·58
		4	$2\frac{3}{8}$	1·98	1·91	2·41	2·14	3·07	2·18	2·28
		8	$2\frac{3}{8}$	1·78	1·52	2·67	2·17	2·00	1·03	1·86
11/4/49	0915 to 1445 G.M.T 13/4/49	4	4	1·16	1·41	0·90	1·27	1·36	1·15	1·21
		4	$2\frac{3}{8}$	0·97	0·69	1·22	0·81	0·57	0·79	0·84
		8	$2\frac{3}{8}$	1·06	1·03	1·20	1·03	1·18	1·09	1·10

A formal analysis of variance carried out on each of the above sets of data leads to the following conclusions as regards significance at the 5 per cent level.

(a) In both sets the differences between corresponding evaporimeters of depths 4 and 8 in. (but of the same diameter) are insignificant. Further-

more, it is to be noted that these differences are generally of opposite signs in the two sets of data, which adds weight to the indication that the differences are chance effects of a natural point-to-point variation in local rate of evaporation.

(b) The differences between corresponding evaporimeters of different diameters (but of the same depth) are insignificant in the first set, but just significant in the second set. In other words there is an indication, though not a very marked one, of a genuine difference in the indications of the evaporimeters of different diameters.

The examination of the effect of depth of evaporimeter strongly suggests that over the period involved the break in soil and root continuity at a depth of 4 in. had no effect on the water loss from the surface. From this point of view, therefore, it is legitimate to accept the hourly measurements carried out with the larger diameter evaporimeters during the same period as indicative of the true water loss. Some uncertainty, however, is now introduced by the suggestion that the evaporimeters of smaller diameter indicate systematically lower than those of larger diameter. A tentative explanation of this effect is as follows. Although it may reasonably be assumed that soil and root continuity were preserved throughout the main bulk of the soil under test, it is evident that this condition would not be maintained right up to the outside of the core. As already mentioned superficial cracks seemed to be unavoidable and it is clear that cutting of some of the roots near the outside would also occur. Thus, around the edge of the soil core, discontinuities in root and soil could have existed sufficiently near to the surface to reduce the local water loss. Such an 'edge effect' would presumably exert a proportionately greater effect with decrease of core diameter and so lead to differences in the direction observed. If this explanation is correct then it implies the possibility that the losses from even the larger diameter evaporimeters are underestimates of the true evaporation loss, but to an extent which it is not possible to estimate from the present data alone. Indeed in view of the rather slender evidence for the effect it is clear that a determination of its precise magnitude would require a much more comprehensive test than has been provided here. In the following considerations of the results, however, it will be seen that the general consistency demonstrated suggests that the effect is of a small order.

4. Profile data and estimation of rate of evaporation

The data on wind velocity, temperature and vapour-pressure profiles are presented, with other relevant data, in Table II. They are divided into two groups, (labelled E and F, following the grouping of the earlier data), corresponding to the two separate sets of evaporimeters. Observations 7 and 8 are omitted since owing to an accident with the evaporimeters no reliable measurements of water loss were obtained. The vapour-pressure differences (Δe) for the various height intervals were evaluated from a specially prepared chart based on the formula

$$\Delta e = \beta\, \Delta T' + A\,(\Delta T' - \Delta T) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

TABLE II. Profile and evaporation data

Group		E					F						
Date		23 March 1949				12 April 1949	13 April 1949						
Observation Number		1	2	3	4	5	6	9	10	11	12	13	14	15
G.M.T.*		1330	1430	1530	1730	1830	1130	1000	1100	1200	1300	1400	1530	1630
Wind direction		ESE.	ESE.	ESE.	ESE.	ESE.	WSW.	W.	W.	W.	W.	W.	WSW.	WSW.
Cloud		6/10 Cu	7/10 Cu	4/10 Cu	1/10 Ac	1/10 Ac	5/10 Sc 9/10 Ac	9/10 Sc	5/10 Sc	5/10 Sc	5/10 Sc	2/10 Cu	1/10 Cu	4/10 Sc
Wind velocity in cm/sec at heights in cm	200	351	330	295	265	189	859	579	621	707	713	702	617	592
	150	328	309	275	243	167	807	543	582	665	672	658	579	553
	100	311	293	259	222	146	757	507	549	625	625	619	546	520
	50	259	243	211	179	108	640	430	461	526	535	525	462	435
	$37\frac{1}{2}$	244	227	199	165	95	601	401	434	499	497	492	435	411
	25	205	193	167	139	77	513	325	353	405	414	401	349	326
Vapour pressure differences in mm Hg. for intervals in cm	200—100	−0·25	−0·21	−0·17	−0·09	+0·05	−0·14	−0·18	−0·27	−0·30	−0·23	−0·26	−0·21	−0·17
	150—$37\frac{1}{2}$	−0·55	−0·45	−0·37	−0·15	+0·12	−0·31	−0·38	−0·58	−0·61	−0·50	−0·51	−0·47	−0·31
	50—25	−0·31	−0·29	−0·22	−0·07	+0·07	−0·17	−0·20	−0·29	−0·30	−0·26	−0·26	−0·25	−0·16
Temperature differences in °F for intervals in cm	200—100	−0·44	−0·27	−0·05	+0·64	+1·58	−0·27	−0·18	−0·44	−0·44	−0·23	−0·23	−0·11	+0·12
	150—$37\frac{1}{2}$	−1·00	−0·59	−0·06	+1·20	+2·95	−0·46	−0·39	−0·92	−0·96	−0·46	−0·48	−0·21	+0·25
	50—25	−0·55	−0·33	−0·02	+0·54	+1·25	−0·23	−0·20	−0·49	−0·51	−0·25	−0·25	−0·12	+0·13
Temperature °F at 200 cm		54·3	56·1	55·7	53·0	49·1	58·9	55·2	57·2	58·9	59·7	61·1	61·9	61·5
Observed rate of evaporation (10^{-5} gm cm^{-2}/sec)		0·60	0·40	0·30	0·12	0·02	0·64	0·50	0·68	0·76	0·63	0·63	0·54	0·39
S for intervals in cm †	200—100	−0·027	−0·019	−0·004	+0·034	+0·084	−0·003	−0·003	−0·008	−0·006	−0·003	−0·003	−0·002	+0·002
	150—$37\frac{1}{2}$	−0·015	−0·009	−0·001	+0·020	+0·058	−0·001	−0·002	−0·004	−0·004	−0·002	−0·002	−0·001	+0·001
	50—25	−0·005	−0·003	0·000	+0·008	+0·032	0·000	0·000	−0·001	−0·001	0·000	0·000	0·000	0·000

* Mid-time of observation period of approximately 1 hr.

† S approximates to half the Richardson Number at the geometric mean height (see Eq. (5)).

where ΔT, $\Delta T'$ are the dry- and wet-bulb temperature differences over the interval, β is the slope of the saturation vapour pressure-temperature curve at the mean wet-bulb temperature and A is the aspirated psychrometer constant. This follows immediately and with sufficient accuracy from the usual psychrometric formula when the variation of wet-bulb temperature over the height interval is not more than a few degrees. For example, with wet-bulb temperatures in the region of 60°F a variation of 5°F introduces an error of only 0·0025 mm Hg. The observed rates of evaporation are also given in Table II, each figure being based on the mean water loss indicated by the six evaporimeters of 4 in. diameter over a period of approximately 1 hr.

Using most of the profile data we may compute rates of evaporation (E_c) from a formula the basis and derivation of which have been discussed previously (Pasquill 1949 a). The formula is

$$E_c = \frac{\rho k^2 (u_2 - u_1)(q_1 - q_2)}{\left[\log_e \left(\frac{z_2 - d}{z_1 - d}\right)\right]^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where u_1, u_2, and q_1, q_2 are wind velocities and specific humidities at heights z_1, z_2, above the soil surface, k is von Kármán's constant of magnitude 0·4, ρ the air density, and d a " zero point displacement " (see later). In a form suitable for direct evaluation from the present data this becomes

$$E_c = \frac{4{\cdot}63 \times 10^{-5} (u_2 - u_1)(e_1 - e_2)}{T \left[\log_e \left(\frac{z_2 - d}{z_1 - d}\right)\right]^2} \quad . \quad . \quad . \quad . \quad (3)$$

where E_c is in gm cm^{-2}/sec, u in cm/sec, e in mm Hg., and T is the dry-bulb temperature in °A. The error involved in neglecting the variation of T over the heights involved is obviously negligible.

Basically the above formulae are valid only for flow over an aerodynamically rough surface in " adiabatic " conditions, *i.e.* in conditions of zero vertical gradient of potential temperature. In the earlier investigation, however, it was demonstrated that the effective departure from adiabaticity is specified, not by the vertical gradient of potential temperature alone, but by the Richardson Number

$$\mathrm{Ri} = \frac{g}{T} \frac{\left(\frac{\partial T}{\partial z} + \Gamma\right)}{\left(\frac{\partial u}{\partial z}\right)^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Γ being the dry-adiabatic lapse rate. Conditions are effectively adiabatic if the Richardson Number is numerically small and hence may exist even in the presence of a substantial temperature gradient provided this is associated with a large vertical gradient of wind velocity. In the present analysis the departures from effectively adiabatic conditions are conveniently examined on the basis of a pseudo Richardson Number, S, evaluated from the formula

$$S = \frac{(\theta_2 - \theta_1)}{(u_2 - u_1)^2} \log_e \left(\frac{z_2 - d}{z_1 - d}\right) \sqrt{(z_2 - d)(z_1 - d)}, \quad . \quad . \quad (5)$$

where θ is the dry-bulb temperature in °F and d has the same meaning as before. At normal temperatures S corresponds to approximately half the Richardson Number at the geometric mean of the two heights, and follows directly from (4) by assuming a logarithmic variation with height of temperature and wind speed and neglecting Γ. The reason for evaluating (4) at the geometric mean height of the interval is that the errors due to actual departures from a logarithmic variation will be minimised.

For the evaluation of the above formulae we require, in addition to the data already specified, the value of the zero point displacement, d. This is originally introduced in the "wind profile-drag" relationship on which Eq. (2) is based, and may be said to take into account the effective raising, by the vegetation cover, of the boundary from which turbulent exchange is operative. The "wind profile-drag" relationship in adiabatic conditions may then be written

$$u = \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}} \log_e \left(\frac{z - d}{z_0}\right), \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where τ_0 is the aerodynamic drag at the surface, and z_0 is the so-called roughness parameter, the magnitude of which depends on the size, shape and disposition of the roughness elements. The value of d may be evaluated graphically in terms of wind velocities at three or more heights, since from (6),

$$\frac{(u_2 - u_1)}{(u_4 - u_3)} = \frac{\log (z_2 - d) - \log (z_1 - d)}{\log (z_4 - d) - \log (z_3 - d)} \quad . \quad . \quad . \quad . \quad (7)$$

A similar equation in terms of vapour pressure necessarily follows from (3), since in deriving (2) and (3) it is assumed that the transport of vapour and momentum follow the same law.

During the observations of 23 March 1949 the vegetation consisted of grass and young clover 1-2 cm high as a basic cover, with frequent patches 5-10 cm and occasional grass stalks 10-20 cm. By 12 April the basic cover was 1-5 cm, frequent patches about 10 cm and occasional stalks 15-20 cm. Preliminary evaluation of S in Eq. (5) indicated that observations 3, 6 and 9-15 could be included in a suitable near-adiabatic category and could therefore be employed in evaluating d from Eq. (7) and the corresponding equation in terms of vapour pressure. The values have been computed for the lower (50-25 and 150-37½) and higher (200-100 and 150-37½) pairs of intervals, and are reproduced in Table III below.

TABLE III. ZERO POINT DISPLACEMENT d IN WIND VELOCITY AND VAPOUR-PRESSURE PROFILES
(Values from vapour-pressure profiles in brackets)

Observation Number	d in cm	
	Lower Interval	Higher Interval
3	8 (9)	7 (10)
6	11 (6)	0 (12)
9 — 15	16 (2)	0 (4)

The values for observation 3, though slightly higher than might have been expected, are mutually consistent. In the remaining cases, however, the wide variations indicate considerable deviation from the logarithmic form of profile, particularly in the case of wind velocity. This is also immediately evident from a graphical examination of the wind profiles; for observations 6 and 9-15 the 25-cm wind velocity seems unduly low compared with the rest of the profile. It seems probable that this is a chance effect due to the rather patchy vegetation cover, a feature which will be discussed later. For immediate purposes it appears that little reliance can be placed on the individual values of d given above. The most consistent values (observation No. 3) and the visual impressions of the grass cover suggest that d should be somewhat below 10 cm. Fortunately a variation of a few cm at this range is of little practical importance in considering profiles over the height range involved here and it is therefore proposed to adopt a round value of 5 cm. The agreement then obtained with the observed wind velocity profiles is illustrated in Fig. 2 where it will be seen that the only important discrepancy occurs at the 25 cm point in observations 6 and 9-15.

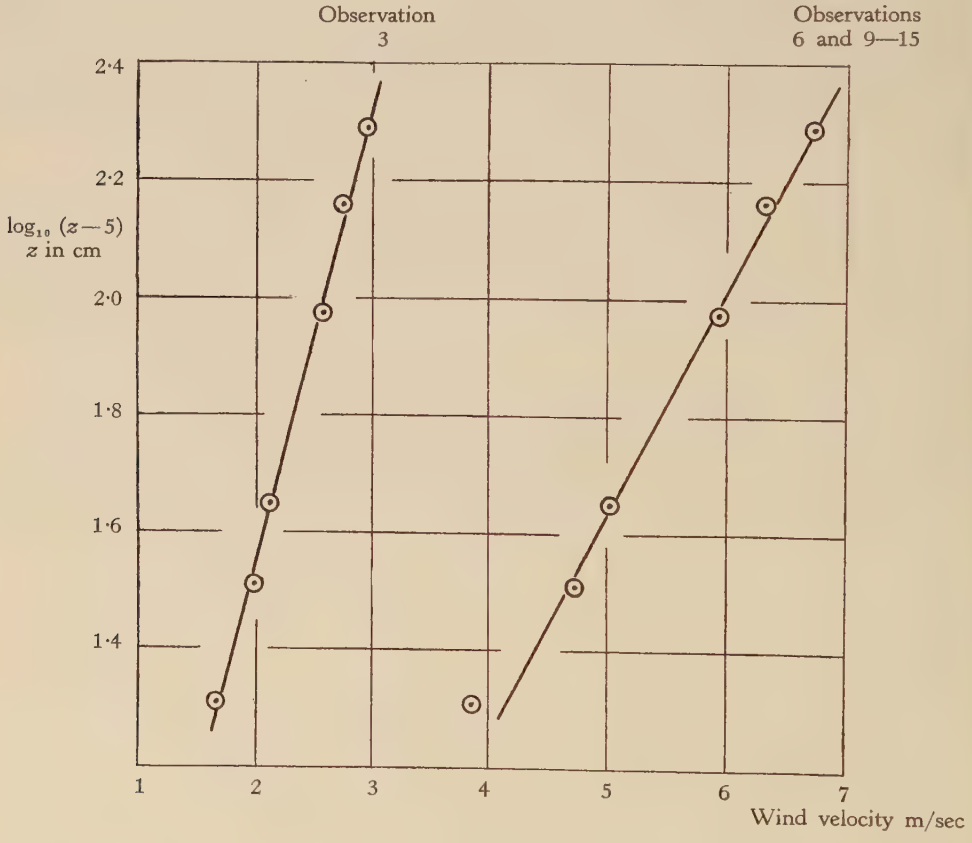


Figure 2. Vertical profiles of wind velocity over grassland.

The final values of the parameter S are given in Table II for each of the three height intervals and on the basis of these we may finally select the cases in which it is appropriate to make comparison with Eq. (3). From the earlier investigation it follows that a suitable near-adiabatic category is specified by $S \leqslant 0{\cdot}004$ numerically. This category, it will be seen, includes practically all the results from observations 6-15 and a proportion of those from observations 1-5. For these cases the ratios of the computed to the observed rate of evaporation (E_c/E_o) have been evaluated and the results are summarised in Table IV below. To these have been added the corresponding results from the earlier investigation. As an indication of the surface roughness the adopted values of d and the appropriate values of z_0 (from Eq. (6)) are also given. For the present series z_0 was evaluated from a $u_{150}/u_{37\frac{1}{2}}$ wind ratio of 1·36, the actual values for observation 3 and 6-15 being respectively 1·38 and 1·34. The lower value for observations 6-15, as compared with that for observation 3, implies a lower degree of surface roughness, despite the rather longer grass actually occurring during observations 6-15. It is to be noted however that the wind velocities were considerably higher during observations 6-15 than during observation 3. A similar effect has been observed previously by Deacon (1949) in much greater detail, and has been ascribed to an effective smoothing of the surface due to deflection of the grass blades in high wind velocities.

TABLE IV. RATIO OF COMPUTED (E_C) TO OBSERVED (E_O) RATE OF EVAPORATION IN NEAR-ADIABATIC CONDITIONS

(a) $z_2 = 200$, $z_1 = 100$ cm.
(b) $z_2 = 150$, $z_1 = 37{\cdot}5$ cm.
(c) $z_2 = 50$, $z_1 = 25$ cm.

Series	Obs. Number	u_{200} cm/sec	E_C/E_O (a)	(b)	(c)
	2	330			0·89
	3	295	0·64	0·68	0·79
March and	6	859	0·69	0·72	0·83
April 1949	9	579	0·81	0·78	1·03
(present obs.)	10	621	—	0·91	1·13
$d = 5$ cm	11	707	—	0·96	1·17
$z_0 = 0{\cdot}51$ cm	12	713	1·00	1·00	1·22
	13	702	1·06	0·96	1·25
	14	617	0·85	0·90	1·27
	15	592	0·97	0·81	1·09
	18	422	0·82	0·83	1·01
	21	479	—	—	1·24
	22	490	—	1·21	1·20
March 1948	24	522	—	—	1·11
$d = 0$	25	570	—	—	1·43
$z_0 = 0{\cdot}25$ cm	26	525	—	1·12	1·11
	27	376	1·24	1·11	—
	29	781	—	0·86	0·86
	30	654	—	0·86	—

The above data present a total of 41 comparisons of observed and estimated rates of evaporation over periods of approximately one hour, based on 19 sets of measurements of rate of evaporation and covering a wind velocity range of 3-9 m/sec. Summarising ;

Arithmetic Mean $E_C/E_O = 0{\cdot}99$

Deviation of E_C/E_O from 1·0	Percentage of cases
$< 0{\cdot}1$	24
$< 0{\cdot}2$	66
$< 0{\cdot}3$	90

These deviations seem to be mainly casual in nature, the only systematic trend being in the generally higher order of the present values of E_C/E_O based on the 50 - 25 cm interval as compared with those based on the higher intervals. This is obviously associated with the previously discussed wind-profile distortion.

It will be noted that the data of Table II include a number of cases in which the magnitude of the parameter S falls outside the range adopted for effectively adiabatic conditions. The cases are in themselves too few to merit detailed discussion, though it has been verified that the evaporation-profile relationship exhibited is broadly consistent with that found in the earlier investigation in which greater deviations from adiabaticity were encountered. An exception occurs in the case of observation 5 where in stable conditions (S positive) the observed evaporation and vapour-pressure difference are of mutually inconsistent signs, implying a transport of vapour against a gradient. It is doubtful, however, that any significance can be attached to this isolated case, particularly in view of the low orders of magnitude of the observed quantities.

5. Reliability of the evaporation-profile relationship

In appraising the ultimate reliability of the preceding results it is important to consider the extent to which the deviations exhibited may in fact be attributed to deficiencies in the observational data, rather than to some genuine departure from the implied fundamental relationship. In testing this relationship we are employing data on evaporation, wind velocity and vapour pressure profiles appropriate to various " samples " of the atmosphere and ground. Apart from instrumental error, the various samples taken may not be mutually representative, owing to local variations of rate of evaporation and surface drag arising from the patchy nature of the ground surface.

To provide some quantitative indication of the resulting deviations observations of wind velocities at 100 and 25 cm and of wet-bulb temperature difference over this height interval were carried out simultaneously at three adjacent positions. The temperature installations were approximately 0·3 m apart across wind, the anemometer installations approximately 2 m apart, the wider separation in the latter case being necessary owing to the forms of the supporting masts employed. Table V below summarises the data, each value being a mean for two 10-min observations, for the second of which the anemometers and psychrometers were interchanged over each height interval. The temperature-difference readings were taken at 1 min

intervals. The grass cover was similar to that occurring during the main series of observations, though somewhat less patchy owing to subsequent grazing.

TABLE V. WIND VELOCITIES AND VAPOUR PRESSURE DIFFERENCES AT THREE CLOSELY ADJACENT SITES

11 May 1949 *Cloud* : 1 — 3/10 small Cu.

G.M.T.	u_{100} cm/sec			u_{25} cm/sec			Wet-bulb temp. differences, (100 — 25) cm, in °F		
1147 — 1217							−1·46	−1·55	−1·48
1230 — 1250	417	421	423	279	281	267			
1310 — 1340	432	432	427	291	295	275	−1·40	−1·47	−1·38
1353 — 1413	482	468	483	329	335	311			
1505 — 1535	405	409	407	271	283	267	−0·90	−0·94	−0·88
1548 — 1608	357	360	365	239	241	231			

The wind velocities exhibit a variation which, as would be expected, is the more pronounced at the lower level. An examination of the component runs, with the anemometers in opposite orders vertically, shows the same variation fairly consistently, suggesting that these variations are for the most part genuine and not due to instrumental error. For the wet-bulb temperature differences a spatial variation is also indicated, though it seems possible that this could be a spurious effect due to the fact that a less complete sample is provided by temperature readings at minute intervals than by the continuous anemometer runs. To test this, readings were carried out at intervals of 10 seconds over 20 minute intervals intermediate with those specified in the table, and the standard deviation of the various possible means, based on one minute intervals of reading, evaluated in terms of the overall mean. The standard deviations were 0·01 or 0·02 °F, suggesting that the variations shown in Table V are to some extent genuine spatial variations.

Now in the conditions of these measurements and of the majority of those considered in the main analysis the vapour-pressure difference depends primarily on the first term of Eq. (1) and hence on the wet-bulb temperature difference. It was therefore possible to infer that the above variations in wet-bulb temperature difference correspond to a variation in vapour pressure of up to $\pm$ 0·02 mm Hg. in a magnitude of about 0·5 mm Hg., *i.e.* about $\pm$ 4 per cent. The wind velocity differences over the height interval likewise show a variation of the order of $\pm$ 10 cm/sec in about 140 cm/sec, *i.e.* about $\pm$ 7 per cent variation. Thus the rates of evaporation based on Eq. (3) and using values of $(u_2 - u_1)$ and $(e_1 - e_2)$ at single positions could vary by as much as $\pm$ 10 per cent. Remembering that there is also likely to be a casual error in the observed rates of evaporation, but of a magnitude which it is not possible to specify from the present data, it is seen that deviations such as those found in the values of E_c/E_o are to be expected. It is also evident that wind-velocity deviations of the above order would explain the apparent distortion at the bottom of the wind profiles in observations 6 and 9-15, (see Fig. 2).

The average value obtained for E_c/E_o (0·99) implies excellent agreement between the present observations and a treatment based on well established laboratory laws for flow over a rough surface. We have to remember, however, that it has not been possible to justify the technique of evaporation measurement completely on *a priori* grounds, though as far as can be judged from the results discussed in section 3 it seems likely that any systematic error could only be in the form of an underestimate. On the other hand the treatment on which the computed values are based has been shown independently to be in good agreement with the observed diffusion of artificially liberated smoke and vapour (Calder 1949). Taken as a whole these features strongly suggest that both observed and computed values are good approximations to the actual rate of evaporation. The practical implication, therefore, is that for surfaces covered with grass up to moderate lengths, and in near-adiabatic conditions of air flow, evaporation over short periods (about one hour) may be estimated from measurements of wind velocity at three heights and vapour pressure at two heights, with a high degree of accuracy on the average. Errors in individual estimations (excepting occasions of very low rates of evaporation) may be expected to be less than those implied by the deviations shown by E_c/E_o, since some part of these is undoubtedly due to casual deviation in the values of E_o. Thus, for example, we could expect errors less than $\pm$ 20 per cent on more than 70 per cent of occasions.

In conclusion it is of interest to consider briefly the validity and practicability of applying the present method of *indirectly evaluating* natural evaporation (including transpiration) to areas under crops, in which cases it is clear that in general the *direct measurement* of evaporation presents even greater difficulties than those encountered with the present selected conditions of soil and vegetation cover. It is to be noted that the aerodynamic law expressed in Eq. (6) was generally established in the laboratory for flow over flat plates bearing roughness elements of different sizes, shapes and dispositions (Schlichting 1936). Aerodynamically the only difference between a surface bearing grass and one bearing crops is a difference in roughness elements and indeed it has been verified (Paeschke 1937) that, in functional form at any rate, Eq. (6) holds over a wide variety of natural and cultivated surfaces including snow, fallow land, grass, wheat and turnips. Thus there is no *a priori* reason why the present method of estimating natural rate of evaporation should not be applied, with equal confidence, to a cropped surface. The only obvious difficulties would appear to lie in the satisfactory specification of the vertical profiles of wind velocity and vapour pressure and it is clear that an exhaustive investigation of such features would be required. For example, measurements at conveniently accessible heights over tall crops would require careful interpretation as regards the ' zero-point displacement.' Crops of large individual size and spacing will presumably impose a pattern on the wind velocity and vapour-pressure fields, so that the representative value of measurements at a single site would need to be carefully investigated. In considering areas of small extent there is the problem of ensuring that the profiles observed are truly representative of the immediate surface. Finally, in the latter connection, it is clear that for small areas, and also

at the upwind edges of larger areas, there will be a progressive development o the profiles as the air flows over the surface, so that caution must be exercised in applying a treatment which assumes horizontally uniform conditions.

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Empirical formulae for the terminal velocity of water drops falling through the atmosphere

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SUMMARY

From a consideration of experimental measurements of the terminal velocity of water drops and of the drag coefficient of rigid spheres it is suggested that a formula of the type

$$V = A \exp bz \left\{ 1 - \exp \left[- (d/a)^n \right] \right\}$$

may be used to represent the terminal velocity of water drops which are larger than those to which Stokes's law is applicable and which are falling through a standard atmosphere. If V (the terminal velocity) is measured in cm/sec, z (the height) in km and d (the drop diameter) in mm the values of the constants are as follows :—

Range of d mm	I.C.A.N. atmosphere				Summer tropical atmosphere			
	A	b	a	n	A	b	a	n
0·3 to 6·0	932	0·0405	1·77	1·147	958	0·0354	1·77	1·147
0·05 to 0·3	191	0·0290	0·316	1·754	188	0·0256	0·304	1·819

If the drops have diameter less than 0·05 mm, Stokes's law is applicable and

$$V = A\, d^2 \exp bz$$

where A is 3040 and 2840 in the I.C.A.N. and S.T. atmospheres respectively and b has the values 0·0191 and 0·0172 in these two atmospheres.

1. Introduction

It is sometimes convenient to have an explicit formula for the terminal velocity of water drops falling through the atmosphere. It is even more convenient if the structure of the formula is sufficiently simple for it to be amenable to mathematical manipulation. Stokes's law for the resistance to motion of a rigid sphere moving in a fluid leads to such a formula but is valid only for very small drops. Schmidt (1909) has proposed a formula for the terminal velocity of raindrops but it was based upon experimental values which do not agree with more modern measurements.

In meteorology the terminal velocity of a water drop is determined by the diameter of the drop (or of the sphere of equivalent volume if the drop is deformed) and by the density and viscosity of the atmosphere. The density and viscosity of the atmosphere vary independently but for many purposes it is adequate to know the terminal velocity of a drop of water falling through a standard atmosphere

in which both density and viscosity are functions of the height. The purpose of the following paragraphs is to propose formulae for the terminal velocity of water drops falling through the I.C.A.N. standard atmosphere and through the standard summer tropical atmosphere. The atmospheric characteristics which have been assumed are shown in Table I. (See *Meteorological Glossary*, 3rd Ed., p. 179, H.M.S.O., London). With the adoption of these characteristics the density and viscosity are to be regarded as functions of height and the formulae for terminal velocity will contain only drop diameter and height as independent variables.

TABLE I. Characteristics of the atmosphere

	I.C.A.N.			Summer Tropical		
Density $gm/cm^3 \times 10^{-4}$	Height m	Temp. °C	Viscosity poise$\times 10^{-4}$	Height m	Temp. °C	Viscosity poise$\times 10^{-4}$
12·251	0	15·0	1·79	—	—	—
11·242	—	—	—	0	41·0	1·91
11·110	1000	8·50	1·76	—	—	—
10·000	2058	1·62	1·72	1311	33·20	1·87
9·105	3000	− 4·50	1·69	2335	25·82	1·84
8·000	4220	−12·46	1·65	3711	16·88	1·80
6·970	5500	−20·75	1·61	5135	7·64	1·75
6·000	6860	−29·58	1·56	6627	−2·07	1·71

2. Experimental data available

Experimental measurements of the terminal velocities of water drops falling through air have been published by Lenard (1904), Schmidt (1909), Flower (1928), Laws (1941) and Gunn and Kinzer (1949). The first three used methods which were inferior in accuracy to that obtainable by more modern techniques. The results of Laws and of Gunn and Kinzer are very similar and differ markedly from those of the other three investigators.

The writer has been permitted to use some unpublished experimental results obtained by C. N. Davies (1942) and quoted in an unpublished report by Sutton. Davies not only measured terminal velocities at normal atmospheric pressure, he also measured the velocities at various reduced pressures. It is not the purpose of this note to describe Davies's work and it need only be mentioned that he measured the velocities by allowing the drops to fall through sharp-edged beams of light thus operating a recording Einthoven fibre galvanometer through the medium of photoelectric cells. A time base was provided by a second fibre operated by a valve-maintained elinvar fork. Davies's results obtained at normal air pressure differ but little from those of Laws and of Gunn and Kinzer. For meteorological purposes the differences between these three sets of velocities are insignificant and, other things being equal, it would be necessary to choose between them or to take average values. However since Davies obtained results at various atmospheric densities it seemed preferable to use only his data in constructing the formulae. From Davies's results terminal velocity has been plotted against air density for a number of drop diameters and the values given in Table II obtained by interpolation.

TABLE II. TERMINAL VELOCITY OF WATER DROPS FALLING THROUGH AIR OF DENSITY ρ (G/CM^3) AND VISCOSITY $1{\cdot}81 \times 10^{-4}$ POISE. (from C. N. DAVIES)

$\rho \times 10^4$	6·00	6·97	8·00	9·105	10·00	11·11	11·242	11·888	12·00	12·251
					Terminal velocity (cm/sec)					
Diam. (mm)										
0·69								278		
0·77								318		
0·79								314		
2·29								697		
3·38	1059	1014	971	929	895	852	849	827	824	816
4·07	1102	1055	1011	970	940	901	898	876	872	866
4·42	1135	1085	1040	994	960	921	917	899	892	885
4·93	1147	1102	1058	1012	978	934	929	902	900	890
5·42	1171	1125	1076	1026	986	944	939	918	914	907
5·95	1200	1142	1086	1032	991	948	941	919	915	906
6·24	1201		1081		988		935	909	908	900
6·37								914		
6·67								917		
7·25								916		

The gaps in Table II at small diameters are due to gaps in Davies's data. The gaps at large diameters have been left because, as will appear later, the formulae proposed do not fit the data at these very large diameters.

It is clear that below a certain size, drops of water will fall as rigid spheres of unit density. For such motion it is customary to define a drag coefficient C by the equation

$$D = C \rho V^2 \pi d^2/8 \quad . \quad . \quad . \quad . \quad (1)$$

where D is the force opposing the motion, ρ is the air density, V the velocity of the sphere and d the diameter. If the right hand side of Eq. (1) is equated to the gravitational force acting on the sphere and if we neglect air density in comparison with drop density (unity) and put $g = 981$ cm^2/sec we can show that

$$C.\ Re^2 = 1308\ \rho\ d^3/\mu^2 \quad . \quad . \quad . \quad . \quad (2)$$

where Re is Reynolds's Number $Vd\rho/\mu$ and μ is the coefficient of viscosity. The merit of this is that the right hand side of Eq. (2) does not contain explicitly the terminal velocity. With this in mind Davies (1945) in a published paper has examined the large amount of experimental information available about the value of C and has deduced the following empirical relationships in which X has been written for $C.\ Re^2$.

$$Re < 4, \qquad Re = \frac{X}{24} - 2{\cdot}3363.10^{-4}\ X^2 + 2{\cdot}0154.10^{-6}\ X^3 - 6{\cdot}9105\ 10^{-9}\ X^4 \quad . \quad . \quad (3)$$

$$3 < Re < 10000, \log_{10} Re = -1{\cdot}29536 + 9{\cdot}86.10^{-1} \log_{10}X - 4{\cdot}6677.10^{-2} (\log_{10}X)^2 + 1{\cdot}1235.10^{-3} (\log_{10}X)^3 \quad . \quad . \quad (4)$$

Thus if the drop size and air properties are known Re can be computed from Eq. (3) or (4) and hence V can be computed for a rigid sphere. We shall use these equations to compute the terminal velocity of drops small enough to fall as rigid spheres.

3. The terminal velocity of large drops

(a) *Variation with diameter*

The values in Table II refer in the main to drops which might be expected to deform from the spherical shape as they fall through the air and it occurred to the writer that the variation of terminal velocity with diameter shown by the values for $\rho = 11{\cdot}888\ 10^{-4}$ g/cm^3 might be adequately represented by an equation of the form

$$V = A_0 \{1 - \exp [-(d/a)^n]\} \quad . \quad . \quad . \quad . \quad (5)$$

where A_0, a and n are constants, V is the velocity in cm/sec and d the drop diameter in mm. By trial and error the values 943, 1·77 and 1·147 respectively were selected for these constants. To obtain a lower limit to the drop diameter for which these values would be appropriate some velocities were computed from Eq. (4). Table III compares values computed from Eqs. (4) and (5) and values from Table II. It is clear that, for meteorological purposes, Eq. (5) with the values of A_0, a and n mentioned, adequately represents Davies's experimental data for diameters 0·69 to 5·95 mm. The comparison between Davies's experimental values for liquid water drops and those computed from Eq. (4) (which is based upon measurement with rigid spheres) strongly suggests that water drops having diameter up to about 1 mm fall as rigid spheres. The agreement between the computed velocities from Eqs. (4) and (5) for diameters less than 1 mm then extends the range of validity of Eq. (5) down to 0·3 mm.

TABLE III. Terminal velocity (cm/sec) of water drops at $\rho = 11{\cdot}888 \times 10^{-4}$ $\mu = 1{\cdot}81 \times 10^{-4}$

d mm	From Eq. (5) $A_0 = 943, a = 1{\cdot}77, n = 1{\cdot}147$	Davies's exptl. values	From Eq. (4) (Rigid spheres)
0·3	115·4	—	114·1
0·5	197·2	—	201·6
0·69	271·4	278	278·5
0·77	301·3	318	308·9
0·79	308·6	314	316·4
1·00	382·0	—	391·2
1·75	591·5	—	613·1
2·29	696·9	697	—
3·38	827·5	827	—
4·07	872·8	876	—
4·42	888·7	899	—
4·93	905·9	902	—
5·42	917·4	918	—
5·95	925·9	919	—
6·24	929·4	909	—
6·37	930·7	914	—
6·67	933·2	917	—
7·25	936·8	916	—

(b) *Adjustment to standard surface conditions*

The values found above for the constants in Eq. (5) refer to density and viscosity values which do not correspond to surface conditions in the standard atmosphere. An average correction for all drop sizes from 0·3 mm to 5·95 mm diameter was made by computing two multiplying factors. One allowed for the difference in air density and was computed from Table II. For the purpose of this correction Table II was extended by computing the terminal velocity of drops with diameter 0·3 and 1·0 mm from Eq. (4). The other factor allowed for viscosity. This second multiplying factor was computed from Eq. (4) by assuming that the effect of viscosity is the same for large water drops as for rigid spheres of equal volume. The product of the two correction factors varied with drop size from 1·003 to 1·015 (mean 1·011) in the I.C.A.N. atmosphere and from 0·989 to 1·029 (mean 1·016) in the S.T. atmosphere. With this correction A_0 now takes the values 932 in the I.C.A.N. atmosphere and 958 in the S.T. atmosphere.

(c) *Variation with height*

The next step was to include in Eq. (5) a term to allow for the variation of the terminal velocity with height. The ratio of the velocity at the various densities in Table I to the velocity at the standard surface density was computed from Table II. Two viscosity correction factors, computed as described above, were applied to allow for the fact that Table II refers to a viscosity of $1{\cdot}81\ 10^{-4}$ poise instead of each column referring to the standard viscosity for the corresponding height. Over the range of heights and drop sizes dealt with in this section this viscosity correction was nowhere greater than 1·3 per cent and over most of the field was substantially less than 1 per cent. The second column of Table IV shows the mean ratio V/V_0 of the velocity at height z to that at the surface for the drop sizes here considered.

TABLE IV. Ratio V/V_0 of velocity at height z to that at surface

Height (m)	Drop diameter (mm)		
	0·3 to 6·0	0·05 to 0·3	< 0·05
	I.C.A.N. atmosphere		
0	1·000	1·000	1·000
1000	1·043	1·027	1·017
2058	1·090	1·061	1·041
3000	1·131	1·090	1·059
4220	1·187	1·130	1·085
5500	1·244	1·175	1·112
6860	1·305	1·227	1·148
	Summer tropical atmosphere		
0	1·000	1·000	1·000
1311	1·051	1·033	1·021
2335	1·090	1·062	1·039
3711	1·142	1·099	1·064
5135	1·198	1·142	1·092
6627	1·256	1·188	1·123

Except at the greatest height tabulated, the error involved in averaging V/V_0 for all the drop sizes dealt with in this section did not exceed 2 per cent. Accordingly in Fig. 1 $\log_{10}(V/V_0)$ has been plotted against z and we see that we may write for heights to 6 km

I.C.A.N. atmosphere	$V = V_0 \exp 0{\cdot}0405\ z$
S.T. atmosphere	$V = V_0 \exp 0{\cdot}0354\ z$

Combining these results with those of the preceding paragraphs we thus get

$$V = A \exp bz \{1 - \exp [-(d/a)^n]\} \quad . \quad . \quad . \quad . \quad (6)$$

where V is the terminal velocity in cm/sec, d is the drop diameter in mm, z is the height in km and, for drop sizes 0·3 to 6·0 mm and heights to 6 km, the constants have the following values.

	I.C.A.N.	S.T.A.
A	932	958
a	1·77	1·77
n	1·147	1·147
b	0·0405	0·0354

4. The terminal velocity of medium-size drops

(a) *Variation with diameter*

The terminal velocity of medium-size drops was next examined to see whether a formula similar to Eq. (6) with different values for the constants, would cover the range from 0·3 mm diameter downwards to the largest drop to which Stokes's law is applicable. From one point of view this might be considered unnecessary since drops of diameter less than 1 mm fall as rigid spheres and their velocities can be computed from Eqs. (3) and (4). These equations however are cumbersome in use. Accordingly Eqs. (3) and (4) were used to compute terminal velocities of drops of various sizes in air having density and viscosity corresponding to surface conditions in the standard atmospheres. For this part of the work these computed values replaced the experimental values of Table II and, since it was possible to choose the air density and viscosity to correspond to the standard atmospheres, no correction factors, such as were used in section 3 (b) for large drops, were necessary.

Having computed V_0 for various drop sizes an equation of the form of Eq. (5) was fitted by trial and error. The range of sizes covered was from 0·3 mm to 0·05 mm ; we shall see later that Stokes's law becomes applicable at this lower limit. The values obtained for the constants were

	I.C.A.N.	S.T.A.
A_0	191	188
a	0·316	0·304
n	1·754	1·819

and the comparison between the values given by Eqs. (3) and (4) and those predicted by Eq. (5) with these values for the constants is given in Table V.

TABLE V. Terminal velocity of medium-size water drops falling through air (cm/sec)
Air density and viscosity appropriate to surface

	I.C.A.N.		S.T.A.	
	Eq. (5)	Eqs. (3) and (4)	Eq. (5)	Eqs. (3) and (4)
d (mm)	$A_0 = 191$, $a = 0{\cdot}316$, $n = 1{\cdot}754$		$A_0 = 188$, $a = 0{\cdot}304$, $n = 1{\cdot}819$	
0·03	3·04	2·72	2·77	2·55
0·04	5·01	4·79	4·65	4·49
0·05	7·37	7·36	6·93	6·93
0·10	23·73	24·96	23·34	23·98
0·15	45·2	46·8	45·5	45·7
0·20	68·9	68·8	70·2	67·7
0·25	92·4	91·3	94·9	90·2
0·30	114·2	113·7	117·4	112·9

It is clear that Eq. (5) adequately represents the terminal velocities of drops within the diameter range 0·05 to 0·30 mm falling through air with the specified properties.

(b) *Variation with height*

The ratio V/V_0 was computed directly from Eqs. (3) or (4) for drops having diameter 0·05, 0·10, 0·15 and 0·30 mm for the various heights in Table I. The mean values for each height are shown in the third column of Table IV. In each case the error involved in adopting a mean value for V/V_0 was greatest for the 0·05 mm drop and varied from 0·8 per cent at 1000 m to 4·5 per cent at 5500 m in the I.C.A.N. atmosphere and from 1 per cent at 1311 m to 3·8 per cent at 5135 m in the S.T.A. $\mathrm{Log}_{10}(V/V_0)$ has been plotted against z in Fig. 1. Over the height range to 6 km we may write,

$$V = V_0 \exp bz$$

where $b = 0{\cdot}0290$ in the I.C.A.N. atmosphere and 0·0256 in the summer tropical atmosphere. Combining these results with those of section 4 (a) we see that Eq. (6) is applicable to medium-size drops (diameter 0·05 to 0·3 mm) at heights up to 6 km provided the constants have the values

	I.C.A.N.	S.T.A.
A	191	188
a	0·316	0·304
n	1·754	1·819
b	0·0290	0·0256

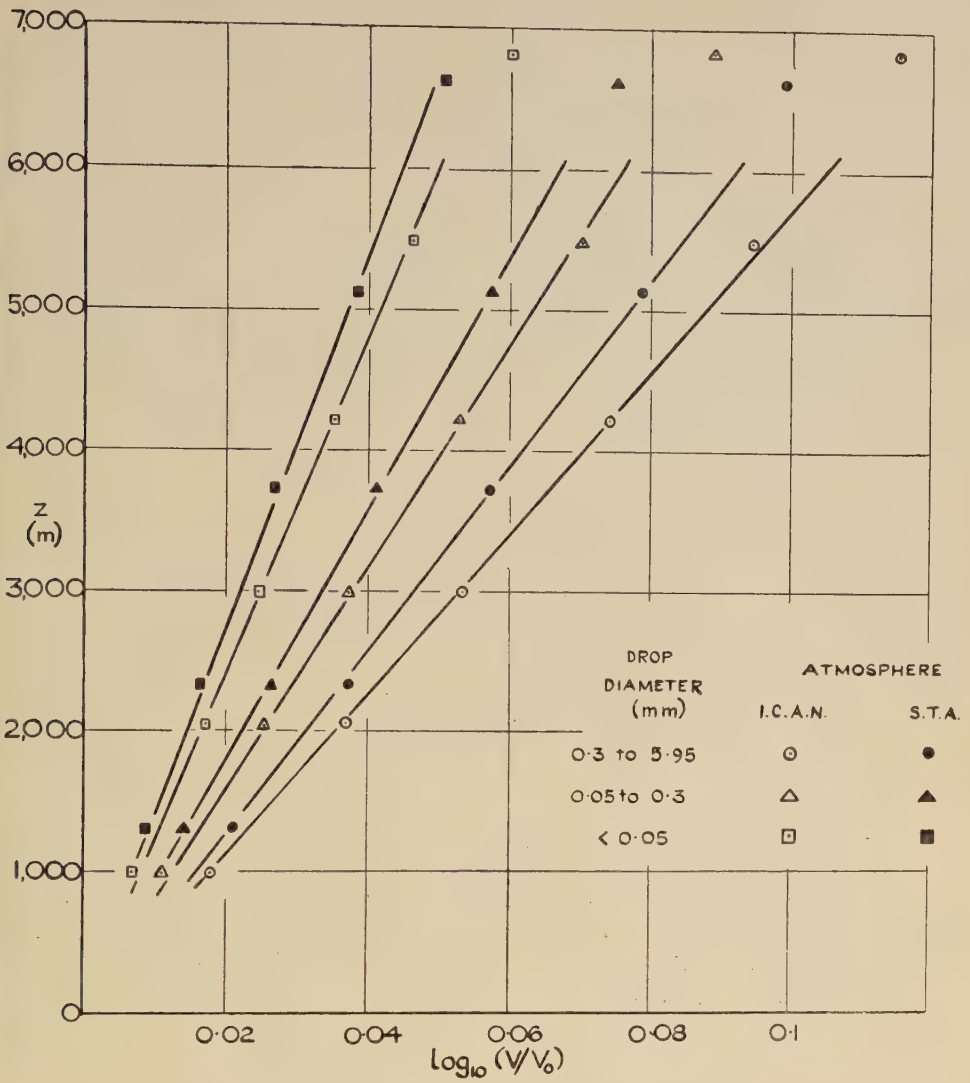

Figure 1. Variation of terminal velocity with height.

5. The terminal velocity of small drops

(a) *Variation with diameter*

The terminal velocity of small drops deduced from Stokes's law is given by

$$V = \frac{g\,(\sigma - \rho)\,d^2}{18\mu} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where σ is the density of the drop. Substituting unity for σ we get

$$V = 3041\ d^2 \qquad . \quad . \quad . \quad . \quad . \quad (8)$$

for $\rho = 12{\cdot}251.10^{-4}$, $\mu = 1{\cdot}79.10^{-4}$, and

$$V = 2844\ d^2 \qquad . \quad . \quad . \quad . \quad . \quad (9)$$

for $\rho = 11{\cdot}242.10^{-4}$, $\mu = 1{\cdot}914.10^{-4}$, where V is measured in cm/sec and d in mm. Table VI shows the terminal velocity of drops computed from Eq. (3) and from Eqs. (8) and (9). If we accept Eq. (3) as giving the true velocity for these very small drops it appears that Stokes's law overestimates the velocity by approximately 3 per cent at a diameter of 0·05 mm and that this overestimation decreases rapidly as the diameter diminishes being less than 1 per cent for a diameter of 0·03 mm.

TABLE VI. Terminal velocity of small water drops falling through air (cm/sec)

d	Stokes's law	From Eq. (3)	Stokes's Law	From Eq. (3)
mm	$\rho = 12{\cdot}251 \times 10^{-4}$ $\mu = 1{\cdot}79 \times 10^{-4}$		$\rho = 11{\cdot}242 \times 10^{-4}$ $\mu = 1{\cdot}914 \times 10^{-4}$	
0·03	2·74	2·72	2·56	2·55
0·04	4·87	4·79	4·55	4·49
0·05	7·60	7·36	7·11	6·93
0·10	30·41	24·96	28·44	23·98

(b) *Variation with height*

It is apparent from Eq. (7) that for drops which obey Stokes's law the variation of V with height is given by the variation of $(1 - \rho)/\mu$. The ratios of this quantity to the surface value for the heights quoted in Table I are given in the fourth column of Table IV and are plotted in Fig. 1. We see that

I.C.A.N. $\quad V = V_0 \exp 0{\cdot}0191\ z$
S.T.A. $\quad V = V_0 \exp 0{\cdot}0172\ z$

Combining these results with Eqs. (8) and (9) we may write for drops with diameter less than 0·05 mm

$$V = A\ d^2 \exp bz$$

where V is the velocity in cm/sec, d the drop diameter in mm, z the height in km (up to 6 km) and the constants have the values

	I.C.A.N.	S.T.A.
A	3040	2844
b	0·0191	0·0172

6. An estimate of the errors

The errors involved in using the formulae set out above are composed of

(a) The experimental errors associated with Table II.

(b) The error involved in fitting Eqs. (5), (8) and (9) to the velocities at the surface.

(c) For the large drops the errors involved in the viscosity correction.

(d) The error involved in assuming mean values of V/V_0 at each height.

Of these (a) can be estimated only approximately but comparison with the results of Laws and of Gunn and Kinzer suggests that 2 per cent would be a safe allowance, and Table III suggests a value not exceeding 2 per cent for (b). Error (c) is scarcely likely to exceed 0·5 per cent and we have seen that error (d) varies from 0 at the surface to about 5 per cent at 6 km. It seems likely then that the errors involved in the formulae may reach 4·5 per cent at the surface and 9·5 per cent at a height of 6 km but that generally they will be much smaller.

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The temperatures at the tops of radar echoes associated with various cloud systems

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SUMMARY

Evidence is produced to support the belief that temperatures at the tops of weather echoes are an indication of the relative strengths of the vertical currents within the echoes. Two types of weather echo are indicated according as the vertical currents are strong or weak, giving support to a theory for two different methods for the production of water drops of raindrop size in the atmosphere.

1. INTRODUCTION

Since 13 March 1947 weather echoes have been observed regularly from the Meteorological Office Radar Station at East Hill, using an A.M.E.S. Type 21 radar equipment which consists of a Type 13 Height-Range Indicator and a Type 14 Plan Position Indicator (P.P.I.) operating on a wave length of 10 cm.

A large number of readings and photographs of weather echoes have been taken showing the horizontal and vertical extent of these echoes in many different weather situations together with, on some occasions, measurements of the intensities of the echoes received.

Other workers in Canada and U.S.A. have been similarly engaged, but, as far as the writer is aware, little has been done in correlating the vertical extent of the weather echoes received with the upper air data provided by a regular sequence of radiosonde ascents at a reasonably close network of stations.

This paper gives the results of the analysis of a large number of height-range photographs and written observations made during the period 13 March 1947 to 4 October 1948.

2. THE EQUIPMENT

The equipment used for height-range measurements is the standard equipment known as A.M.E.S. Type 13 operating on a wave length of 10 cm and using a pulse of 2 μsec duration. A common transmitting and receiving aerial is used which is made up of two sections of a parabolic metal cylinder cut in the focal plane, each section being 20 ft high and 18½ in. wide separated by a 1 in. thick panel. This

gives beam widths (measured to half power) of $7\frac{1}{2}°$ in the horizontal and $1\frac{1}{2}°$ in the vertical plane. The aerial is mounted with the long dimension vertical and is made to sweep continuously about a horizontal axis from —1° to 19°, one complete up and down movement taking 10 sec, while it can be turned to any required azimuth by remote control from the Operations Room.

The presentation on the display tube gives range and height of the echo observed directly, and a simple switching device enables the same tube to be used as an A — scope to give the intensity of the echo examined. This intensity was measured as a signal to noise ratio with the aerial sweeping and hence gives the maximum intensity in the vertical plane comprising the echo at a given range. The characteristics of the radar equipment make it difficult to ensure that the performance of the equipment is absolutely constant from day to day, but a good standard of maintenance ensured that a comparison of signal intensities on different days would at least give the order of the relative magnitude of the signals. Signal intensities quoted in this paper (see Table III) were measured during periods when the performance of the equipment was known, by special tests, to have been particularly uniform and the relative magnitude of the signals on these days will certainly be of the order of those quoted in Table III.

Photographs of the display tube were taken as required on 35-mm film. The principle on which photographs were taken was to endeavour to take a photograph of any interesting echo which was observed. "Interesting" in this connection would not only involve the photographing of the most intense echoes but also echoes of unusual shapes, echoes being investigated by an aircraft under our control and, on some occasions, the following of one particular echo in its changes with time. On many days there were echoes on many bearings while a particular echo might extend through several degrees of azimuth and it was, of course, impossible to photograph all echoes at all stages of their existence. In general, however, a photograph or written record would be available for the most noteworthy echoes, including those extending to the greatest height, of a particular day during the time of operation of the radar (usually to about 1730 clock time). In taking a photograph one particular echo or part of an echo was usually in view and therefore brought directly into the radar beam but this would not preclude the appearance on the photograph of echoes at different ranges which were possibly not directly in the beam (which has a horizontal width of $7\frac{1}{2}°$).

3. Preparation of the readings for analysis

For analysis the ranges at which observations were made (which usually had a maximum value of 75 mi) were split up into the ranges from 11 — 20, 21 — 30, 31 — 40, 41 — 50, 51 — 60 and 61 — 75 mi. Readings were not considered for ranges of less than 11 mi since the maximum angle of the aerial sweep, 19°, did not permit measurements of the highest tops on all occasions at such short ranges.

The readings from the photographs — of which some 1,200 were analysed for this paper — and written records were arranged to show the maximum tops of

weather echoes observed within these ranges, and the maximum top for each day, or for each type of weather if more than one type occurred on one day, was extracted for each range. A close analysis of the weather situation was made for each day and a decision made as to which radiosonde ascent gave the most likely upper air conditions for the locality of the weather echo under examination.

Having decided on the representative upper air ascent for each weather echo the temperature of the air at the height of the maximum tops of the weather echoes was determined for each of the ranges mentioned. Having regard to the difficulties of height measurement on the radar tube — measurements to the nearest 500 ft only being possible — and possible inaccuracies in assessing the representative upper air ascent for the locality in question, it is evident that the temperatures so determined cannot be exact. It is probable that the temperatures are only accurate to within about $\pm$ 3°F. The temperatures so determined were then split up into ranges so that the number of occurrences of tops within a certain range of temperatures at specified ranges of distance was obtained.

On some occasions, however, a particular weather phenomenon would not affect equally all ranges, *e.g.* in thunderstorm conditions an active storm may be viewed at 41 — 50 mi, say, while at 11 — 20 mi only a degenerate storm may be seen with the result that the top of the storm at 41 — 50 mi appears higher than the one at 11 — 20 mi. Similarly in frontal conditions, if the front had already passed the station before observations commenced higher tops might be seen on the longer ranges than on the shorter. On another occasion the distant echo might be fully in the radar beam while the most intense part of a nearby echo might not be on exactly the same bearing giving again an apparently higher top on the longer range. When allowance is made for the fall-off of echo intensity with range R (inversely proportional to R^2 if the echo fills the radar beam and to R^4 if the echo fails to fill the beam) it seems virtually certain that an echo with a high top at long range would have given a top at least as high, and possibly higher, at shorter ranges, so that the typical top for the weather situation in question may not have been directly measured at all ranges. For this reason Table I has been constructed which shows the number of occurrences when the minimum temperature of the top of a weather echo in a particular weather situation, irrespective of the range, occurred within various ranges of temperature ; the number of different weather situations involved is 164.

It is of considerable interest to determine whether the temperature at the tops of weather echoes is any guide to the type of weather associated with the echoes ; *i.e.* whether similar types of weather give echo tops in the same range of temperatures. For this reason the occasions within each temperature range of Table I were examined and the types of weather associated with them listed.

The use of temperatures, in preference to actual heights above M.S.L., to which echoes extended is made in order to afford a basis for comparison between echoes observed at widely differing times of the year or in air with widely differing air mass characteristics. For this reason the emphasis throughout this paper is on temperature rather than vertical extent.

TABLE I. Number of occurrences of minimum echo top temperatures within certain intervals of temperature

Temperature interval (°F)	Number of occasions
Over 35	4
35 — 31	10
30 — 26	17
25 — 21	11
20 — 16	12
15 — 11	5
10 — 06	18
05 — 01	10
00 — −04	16
−05 — −09	14
−10 — −14	3
−15 — −19	5
−20 — −24	5
−25 — −29	5
−30 — −34	3
−35 — −39	5
−40 — −44	5
−45 — −49	5
−50 — −54	3
Below −54	8
TOTAL	164

Finally those occasions when it was thought probable that the echo viewed came from a thunderstorm were separated from the occasions when it was almost certain that no thunderstorms were present. These occasions, with remarks, are listed in Table II. On some of these occasions signal intensity readings were also made as well as on occasions when thunderstorms were unlikely to have been present. A comparison of such occasions during periods when the performance of the equipment was known, by special tests, to have been particularly uniform is given in Table III. To enable a comparison to be made of intensities measured at different ranges the relative power of the signals is given as if all measurements were made at 20 mi range with the echo filling the radar beam.

4. The temperature at the top of the echo in relation to the vertical currents within the echo

An examination of the types of weather associated with the various temperature ranges of Table I brings out the probability of a close relationship between the temperature at the top of the echo and the vertical currents likely to be experienced within the echo.

Vertical currents are generally assumed, with considerable justification, to be large in cumulonimbus cloud and particularly in thunderstorm clouds and to be small, or at any rate if strong of very limited extent, in the layer type of clouds. It can be seen from Table II that a very high proportion of the very cold echo tops are associated with thunderstorm conditions. Thus when the temperature at the

TABLE II. The occasions in the various temperature ranges when thunderstorms were probably viewed

Temperature interval (°F)	Frequency	Remarks
Over 30	1 out of 14	This was observed at 61 — 75 mi and only report of thunder was " thunder heard " at Portland Bill.
30 — 21	1 out of 28	This was at 61 — 75 mi range, thunder reported at Manston.
20 — 11	Nil out of 17	
10 — 01	4 out of 28	1 at 51 — 60 mi, 1 at 41 — 50 mi, 1 may have missed highest tops as obs. only made in morning and 1 case when top was known *not* to have been at a lower temperature.
00 — −09	4 out of 30	On 3 occasions thunder only reported at one distant station (more than 50 mi away) and on remaining occasion obs. were incomplete.
−10 — −19	3 out of 8	1 at 41 — 50 mi, on the other two occasions obs. were concentrated on one area due to other work and may have missed highest tops.
−20 — −29	5 out of 10	1 at 41 — 50 mi, 3 at 31 — 40 mi, 1 at 21 — 30 mi but on this occasion obs. concentrated on one area and almost certainly missed actual storms.
−30 — −39	5 out of 8	1 at 41 — 50 mi and remainder at 21 — 30 mi.
−40 — −49	8 out of 10	The remaining two occasions gave heavy squally showers locally and may have been thunderstorms at the time of observation.
Below −49	11 out of 11	

echo top was below −40°F, 19 out of 21 occasions were almost certainly associated with thunderstorms, while the remaining two were associated with conditions which gave heavy squally showers locally and may well have been thunderstorms at the time of observation. In the range from −30°F to −39°F, five occasions out of eight gave thunderstorms but the ranges at which they were viewed were in all cases greater than 21 mi and, with one exception, lower echo top temperatures, sufficient to put them in the range below −40°F, may have been found if it had been possible to view them at shorter range. The exception mentioned was at −39°F and as this was the tropopause temperature at that time echo top temperatures below −39°F could not have occurred. The remaining occasions of probable thunderstorms which gave echo tops in the ranges with temperatures warmer than −30°F were all viewed at ranges greater than 30 mi or the observations were incomplete for various reasons, as detailed in Table II, with one very important exception.

TABLE III. SIGNAL INTENSITIES ON THUNDERSTORM OCCASIONS COMPARED WITH OCCASIONS WHEN THUNDERSTORMS PROBABLY NOT PRESENT

A. *Thunderstorm occasions*

Date	Minimum echo temperature (°F)	Range (miles)	Relative power at 20 mi	Remarks
1 June 1948	−40	21	2755	
10 June 1948	−49	15	899	
14 June 1948	−65	23	22350	
28 July 1948	−70	43	1332	
30 July 1948	−68	45	987	
3 Sept. 1948	−46	19	3247	
4 Oct. 1948	07	21	317	Very isolated thunder

B. *Thunderstorms probably not present*

Date	Minimum echo temperature (°F)	Range (miles)	Relative power at 20 mi	Remarks
30 Apr. 1948	−23	26½	60	Convection cloud
29 Jan. 1948	−28	22½	101	Occlusion
25 Aug. 1948	−17	12	2916	Cold front, line squall conditions
29 May 1948	−15	28	500	Medium cloud
4 May 1948	−13	20	2024	Warm front
13 Aug. 1948	−01	29	17	Polar trough
22 July 1948	00	48	1122	See Section 6, para 1
8 June 1948	−02	12	16	Thick upper cloud
3 May 1948	−05	38	126	Convection dying out
17 Apr. 1948	−08	34½	461	Trough
29 Jan. 1948	−04	22	97	Back-bent occlusion
9 June 1948	15	15½	14	Limited convection
28 May 1948	13	15	30	Limited convection
7 Aug. 1948	18	19	90	Layer cloud
16 July 1948	18	12	323	Limited convection
4 May 1948	24	21	132	Slow moving occlusion

On this occasion, in the range from 10°F to 01°F, thunder was reported by two stations at a time when a Spitfire aircraft under our control was making turbulence investigations in the neighbourhood. Reports from the pilot of this aircraft, independently confirmed by other pilots and indirectly from upper air ascents, placed the cloud top very close to the observed echo top, the temperature of which was 07°F. On this one occasion, therefore, echo-top temperatures below −30°F would appear to have been impossible but on all the remaining 41 occasions when thunder was likely to have occurred there are plausible reasons for assuming that, if the particular echo had been viewed in the 11 — 20 mi range, the echo-top temperature may well have been lower than −30°F and probably lower than −40°F.

There seem strong grounds, therefore, for assuming that if an echo top is observed at a range of 11 — 20 mi, in this area with this equipment, with a temperature at or below about −40°F then the echo will be from a thunderstorm while

the converse that a thunderstorm, if viewed at 11 — 20 mi, will give an echo extending higher than the —40°F isotherm will very probably be true although exceptionally a much lower height may be observed. Deductions of thunderstorms on the basis of echo tops extending above the —40°F isotherm have been rather strikingly confirmed on one or two occasions when thunder had not been expected.

Conversely, of the 54 occasions when the temperature at the echo top was above 15°F, 39 occasions were listed as belonging to layer type characteristics of cloud, including in this category weak or slow moving or quasi-stationary fronts. Of the remaining 15 occasions, seven, all with temperatures at the echo top less than 32°F, were associated with convection limited by isothermal or stable layers at or near the echo-top temperature or convection dying out ahead of a warm front ; four were not considered because of the great range at which they were viewed and the remaining four referred to two occlusions and two warm fronts.

Considering next the range from 10°F to —09°F, it was found that of 58 occasions eight had already been discussed under the thunderstorm occasions. Of the remaining 50 occasions, 35 were associated with fronts or troughs, the cold fronts and troughs (which are frequently almost indistinguishable from cold fronts) with one exception having echo-top temperatures of 00°F or lower. This exception, when the echo-top temperature was 09°F, was associated with a cold front which the P.P.I. display showed to consist of three narrow parallel belts of precipitation suggesting that the front really consisted of three separate stages of cold air and was therefore presumably not as active as a normal cold front. Warm front phenomena were, in general, associated with echo-top temperatures of 00°F or higher and, as might perhaps be expected, echoes associated with occlusions were more or less evenly distributed through the whole range from 09°F to —09°F. The 15 other occasions in this range which remain to be accounted for contain ten occasions of either limited convection or decaying cumulonimbus activity, two occasions of snow in a cold easterly current, two occasions of warm air instability and one occasion which is dealt with later under Section 6.

Finally in the range from —10°F to —29°F there were 18 occasions of which eight were discussed under the thunderstorm occasions. Of the remaining 10 occasions, four refer to cumulonimbus, two to cold fronts, two to occlusions and one each to unstable medium cloud and a warm front. As the majority of echoes associated with frontal occasions have temperatures at their tops in a higher range of temperatures than this, it can be assumed that the occasions here encountered were those of particularly active fronts in which, owing to instability in the warm air, cumulonimbus clouds were probably embedded in the frontal cloud system.

It is noteworthy that there is an apparent minimum in Table I in occurrence of echo-top temperatures in the range from 11°F to 15°F and this point will be returned to later (Section 5). This minimum was also visible in the occurrences of echo-tops in the various ranges up to 50 mi. Of the 5 occasions in Table I when the minimum echo-top temperature irrespective of range fell in the temperature range 11°F to 15°F four were connected with inversions or isothermal layers at the echo-top heights, indicating limited cloud development, and the other occasion

was associated with layer cloud. It is appreciated that the difficulties of accurately assessing the temperature at the top of the echo (an accuracy of better than $\pm 3°F$ is probably unlikely) may cast some doubt on the reality of this low frequency but if, from the 164 occasions of Table I, we extract the occasions of layer clouds, fronts and troughs (*i.e.* excluding cumulonimbus cases) we can plot a graph of number of echo tops with temperature T°F or higher against T as shown in Fig. 1.

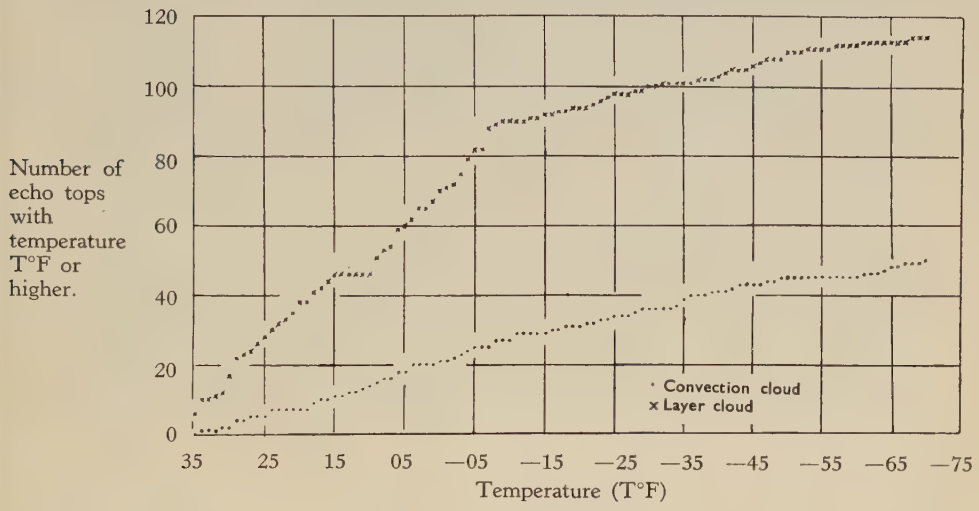

Figure 1. Distribution of echo tops with temperature for convection and layer clouds.

This graph appears to exhibit a genuine discontinuity in the temperature range from 15°F to 10°F. A marked change of slope in this curve is also visible at —09°F and lends force to the suggestion made above that the normal frontal cases have echo tops warmer than —09°F. The similarity of slope of the curve in Fig 1 at temperatures lower than —09°F to the curve for cumulonimbus echoes alone, also shown in Fig. 1, suggests that the cases with echo tops colder than —09°F contained, as suggested above, cumulonimbus clouds embedded in the frontal cloud system.

We can, therefore, construct a rough table (Table IV) showing the temperatures to be expected at the top of weather echoes viewed with this equipment in this area at 11 — 20 mi for various types of weather.

Table IV indicates that in general terms it is almost certainly correct to say that the temperature at the top of the weather echo gives a good indication of the strength of the vertical currents within the weather echo. Absolute values of the vertical currents cannot, of course, be inferred (but see Section 5).

5. Types of weather echo and their probable explanation

Not only, as shown in Table IV, does the echo-top temperature indicate a type of weather likely to be associated with the echo but the echoes themselves, as viewed on the height-range display tube, have certain outstanding characteristics of form and structure.

TABLE IV. TYPES OF WEATHER ASSOCIATED WITH RANGES OF ECHO-TOP TEMPERATURES WHEN VIEWED AT 11 — 20 MILES

Temperature interval	Weather type		
Lower than −40 °F	Thunderstorms		
−40 °F			
−39 °F — −10 °F	Particularly active fronts probably containing cumulonimbus embedded in the frontal cloud systems but probably without thunder. Cumulonimbus clouds probably without thunder.		
−09 °F — 00 °F	Unstable medium cloud. Normal cold fronts and troughs	Most occlusions Snow showers	Convection limited by stable layers at or near the echo top temp., decaying cumulonimbus activity with approach of frontal conditions.
00 °F — 09 °F	Normal warm fronts		
10 °F — 15 °F			
16 °F — 32 °F	Layer-type clouds including weak and slow moving or quasi-stationary fronts.		
Higher than 32 °F			

Let us consider first the occasions when the echo-top temperature is higher than 15°F. The responses on these occasions which, as shown in Table IV, are associated with layer type clouds including weak and slow moving or quasi-stationary fronts are usually in the form of a band of echo of limited vertical development (Jones 1949), rarely giving responses down to ground level and sometimes associated with the brighter band near the freezing level (see Fig. 2).

It has been shown theoretically (Ryde 1946) that if snow falls as dispersed dry ice crystals each having a mass less than about 1 to 2 mg the echo will be less than from rain having the same precipitation rate. It is therefore possible to visualise an occasion of precipitation when the echo is not visible as long as the ice crystals forming the precipitation are discrete, dry and small but becomes visible when something occurs either to increase the size or reflecting power of the ice crystals or to aggregate them into snowflakes or to melt them into raindrops.

The fact that the responses with echo tops warmer than 15°F rarely extend to ground level would indicate that the response from the raindrops was itself too small to be seen by this radar. As the responses from discrete dry ice particles are likely to be less than from rain at the same precipitation rate, it is unlikely that the echo up to 16°F can be due to dry ice particles and the inference is that the precipitation was made visible either by the aggregation of the ice crystals to form snowflakes or by an increase in the reflecting power of the ice crystals. An increase in the reflecting power of an ice crystal will occur if the surface of the ice crystal becomes wet as, for example, in the initial stages of melting as it falls through the

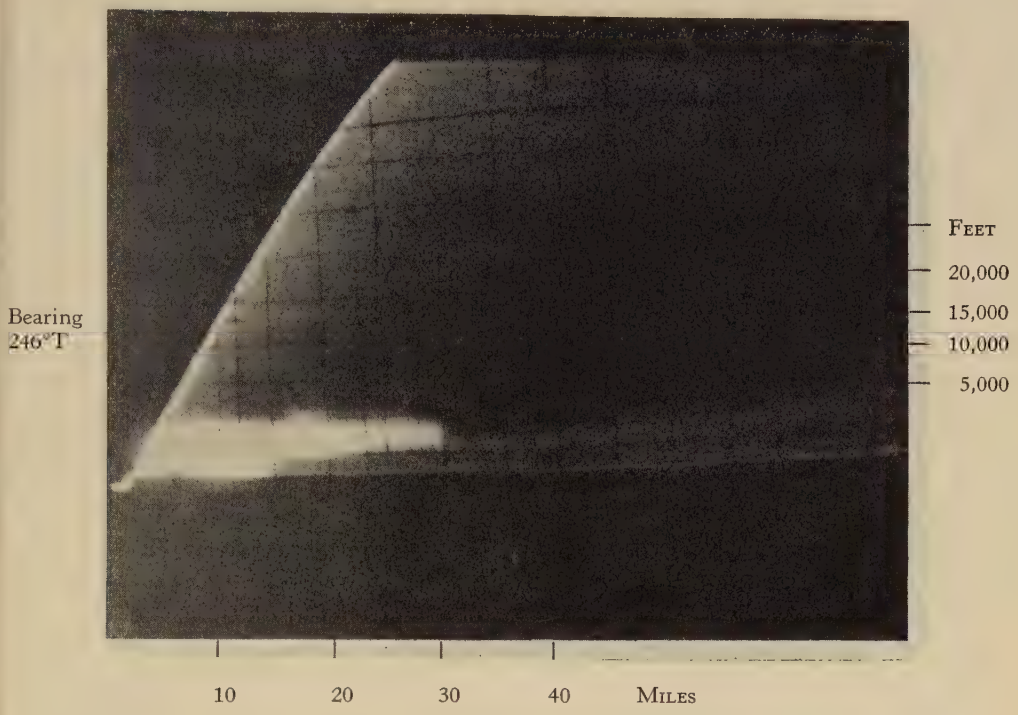

Figure 2. 29 January 1948, 10.44 G.M.T. This is a typical layer type of echo associated with an occlusion. Note the level top extending over a considerable range. The freezing level was at 4,200 ft and at 5,500 ft the temperature was 26°F.

In accordance with the theory of Section 5, this echo is typical of those associated with small vertical currents in which the echo is received from snowflakes and melting snowflakes and has a top warmer than 15°F.

freezing level, leading to the phenomenon commonly known as the "Bright Band" effect (Ryde 1946). On some occasions indeed it is probable that only the enhanced echo due to the bright band effect was visible. The ice crystal surface, however, may become wet at lower temperatures than the melting point if the ice crystal is in collision with numbers of small supercooled water droplets, the latent heat of fusion liberated being sufficient to keep the surface of the crystal at or near 32°F. This can only take place (a) if the crystal collides with a sufficient number of supercooled water droplets and (b) if the temperature is high enough so that the loss of heat to the surrounding atmosphere is compensated by the gain of heat from the latent heat of fusion liberated by part of the supercooled water droplets on freezing. Aggregation of the ice crystals to form snowflakes also probably takes place when the crystals are made wet in this manner (Sutcliffe 1940) so that an increase in echo intensity may be expected to be due to a combination of two effects, the increase due to the ice crystal surface becoming wet and the increase due to aggregation of the ice crystals. It is suggested that with low vertical current

speeds conditions (a) and (b) above are satisfied at temperatures higher than 15°F but that at lower temperatures the ice particles, although growing by collision and condensation in the atmosphere which is supersaturated relative to ice, remain dry.

It can be assumed therefore that responses with echo tops warmer than 15°F are due to wet ice crystals or snowflakes composed of an aggregate of ice crystals or to melting ice crystals in the vicinity of the freezing level. The normal size of individual ice crystal encountered (Hooper and Kippax (1950) quoting Nakaya and Terada) has a terminal velocity of the order of 1/3 to 1 m/sec so that it may be noted that should the vertical current exceed about 1 m/sec no ice crystals can fall past the 16°F isotherm. The explanation given above therefore can only apply to vertical currents of less than about 1 m/sec and probably less than 1/3 m/sec.

We come now to echoes with tops colder than 10°F. As Table IV shows these can be divided into two sections, the one including normal fronts and high level instability and the other cumulonimbus (including particularly active fronts). The normal frontal cases are in the higher part of the range, *i.e.* with temperatures at the top of the echo from 09°F to −10°F and with the cold fronts in the 00°F to −10°F part of this range. These frontal cases were often associated with a bright band indicating the presence of melting snowflakes or ice crystals and hence it would be reasonable to expect a similar explanation to that for echoes with tops warmer than 15°F to apply. If this is so we must assume that (a) aggregation to snow flakes or wet crystal surfaces occur at temperatures as low as −10°F in occasions of stronger vertical currents and that (b) although the vertical currents are stronger they are still less than about 1 m/sec. Examining (b) more closely, the slope of a warm front is usually taken as between 1 in 100 and 1 in 150 (*Met. Glossary*, 1939). The ascent of warm air overriding cold air is the agency producing the rain and the weather echo. If the difference in components of the horizontal velocity of the air at right angles to the front in the warm and cold air is 40 mi/hr (which would certainly be an extreme case) the rate of ascent of the warm air (Brunt 1939) for a slope of 1 in 100 would be about 0·18 m/sec and well within the limit set by (b). In a cold front with a slope of about 1 in 50 the rate of ascent would be greater but still well within the limit set by (b). Bannon (1948) by considering the rate of rainfall also concludes that in active depressions over the North Atlantic the rate of ascent of the air is generally of the order of 10 — 20 cm/sec, so that upward velocities of less than 1 m/sec can be generally said to be associated with all normal frontal cases. Occasionally, however, latent instability in the warm air or other effects may cause the rate of ascent to be increased leading to a cumulonimbus type of echo (see below).

Echoes of the frontal type, therefore, can be explained in the same way as the layer type echoes with tops warmer than 15°F if we can assume that the greater vertical currents (but still less than 1 m/sec) lead to aggregation of the ice crystals to snowflakes or wetting of the crystal surface at a greater height (*i.e.* lower temperature). This might occur since the greater vertical current would slow down the rate of fall of the ice crystal giving it a greater chance of growth and may also

make available a greater number of supercooled water droplets. This explanation however makes it difficult to account for the apparent low frequency of echo-tops in the temperature range of 15°F to 10°F (see Fig. 1). The foregoing explanation would imply that aggregation to snowflakes or wetting of the ice crystal surface is uncommon at temperatures from 15°F to 10°F which itself implies that ice crystals falling from above either pick up enough supercooled water before reaching the 10°F isotherm or, having failed to do so by that time, they must for some reason wait until reaching at least the 16°F isotherm before doing so. An alternative explanation was therefore sought to account for the low frequency of echo tops in the 10°F to 15°F region and a possibility was found in an alternative explanation of the echoes with tops colder than 10°F.

The fact that a bright band is often still observed with such echoes indicates almost certainly that the change of state from ice to water is taking place at or near the freezing level and therefore the probable explanation of the part of the echo above the freezing level must still require the echo to come from the ice phase. The only alternative to the previous explanation is that the echoes come from the dry ice crystals themselves. It has been indicated (Ryde 1946) that when snow falls as dispersed ice crystals, each having a mass of about 1 to 2 mg, we should expect that the echo intensity would not be very different from that produced by rain at the same precipitation rate. With the stronger vertical currents, still less than about 1 m/sec however, and greater vertical extent of cloud in frontal cases than in the other layer type cases giving echo tops warmer than 15°F, the ice crystals descend more slowly and traverse a greater depth of cloud and so are much more likely to reach the 1 — 2 mg mass required to give an echo on their own account although they may, of course, aggregate to snowflakes or become wet in the same way as smaller crystals when falling below the 15°F isotherm.

We therefore have two types of bright band echoes :

(i) In very weak vertical currents, probably less than 1/3 m/sec, and also probably with limited thickness of cloud, echoes, the tops of which are warmer than 15°F, due to the aggregation of wet ice crystals or to the melting of small ice crystals which are not big enough to give an echo on their own account.

(ii) In stronger vertical currents, but less than 1 m/sec, usually associated with frontal systems and probably great depth of cloud, echoes, the tops of which are colder than 10°F, due to the ice crystals themselves, the ice crystals having grown to the requisite size due to their small rate of fall through a considerable depth of cloud.

The low frequency of echo tops associated with a bright band in the range 10°F to 15°F may then be ascribed to a difference in cloud conditions between (i) and (ii) above. In active frontal cases with great depth of cloud and slow rate of fall, the ice crystals have already grown to a size great enough to give an echo on their own account before falling to the 10°F isotherm but in weaker frontal cases and layer cloud the ice crystals are never big enough to give an echo on their own account but must wait until passing the 15°F isotherm before they pick up sufficient

supercooled water to give them a wet skin and also make aggregation likely.

Finally we encounter the cumulus and cumulonimbus type of echo which provides the majority of the echoes having a temperature at the top below —10°F although, as Fig. 1 shows, such echoes can also occur with temperatures at the top at any value below 35°F. At the higher temperatures the echo-top temperature was usually that of the inversion or stable layer limiting the vertical extent of the convection cloud, or very close to it.

The outstanding characteristic of this echo type is its appearance as a strong vertical column of echo (Jones 1949) with clear vertical edges and no indication of any bright band phenomena (see Fig. 3). The strongest part of the echo is *not* in the form of a horizontal band but appears, when the receiver gain is reduced, as a vertical column with the maximum intensity at some point or at a series of points at different heights within this column. The conclusion seems inescapable that we are dealing with a completely different type of echo formation, while the absence of any sign of bright band leads to a suggestion that there is no change in intensity of the echo at the freezing level due to melting because there

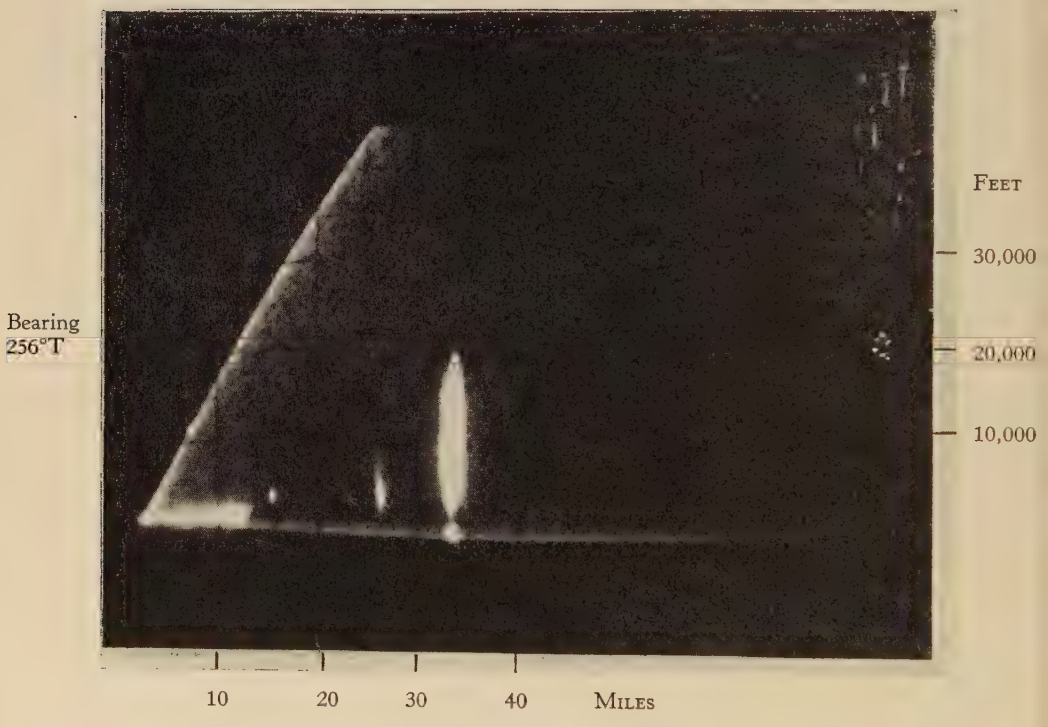

Figure 3. 30 May 1949. 13.48 G.M.T. This is a typical cumulonimbus echo with a top at —29°F. Note the clear cut vertical edges indicating a "solid" type of vertical current. The persistence of form of an echo of this type suggests that, although the vertical current is strong, it is not subject to violent changes in value.

is nothing there to melt ; *i.e.* the echo seen on this equipment from a cumulonimbus cloud comes from liquid water drops at all heights. As support for this idea may be quoted an occasion (on 20 April 1948) when the pilot of a Spitfire aircraft investigating cumulonimbus clouds reported icing from 26,000 ft (temperature —39°F) to 18,000 ft (temperature —07°F) which to use the pilot's own words " seemed to be formed by very small water droplets which spread out on the windscreen in the form of very small transparent oval patches. Its thickness was negligible . . . At 10,000 ft to 8,000 ft there was rain . . ." The temperature at 10,000 ft according to upper air ascents was 22°F and at 8,000 ft, 30°F.

Disregarding for the moment the absence of the bright band, the only alternative to the above hypothesis would be that at some stage the echo came from hail instead of water drops since the existence of snowflakes in the large vertical currents of a cumulonimbus seems impossible. As shown by Hooper and Kippax (1950) large snowflakes have a terminal velocity of about 2 m/sec — a value which is almost certainly exceeded in the upward velocity of the air in the most moderate cumulonimbus clouds (Byers and Braham 1948). The echo intensity from dry hailstones however (Ryde 1946) is only 0·18 of that from raindrops of the same diameter for a wavelength of 10 cm so that to explain the radar echo at some height above the feeezing level solely in terms of dry hailstones would imply hailstones of greater diameter than the raindrops in the normal raindrop distribution which gives an echo, and also the same number per m^3. The infrequency with which hail is reported even when the freezing level is near the surface would suggest that hail is not an essential component of shower type precipitation.

One further possibility remains for consideration, the possibility that the echo above the freezing level is received from wet hail *i.e.* the hailstones at some height above the freezing level are in collision with so many supercooled water droplets that the surface of the hailstone remains always wet. In such a case the hailstone would reflect the incident radar energy in the same way as a water drop of the same size and the difficulty due to the decreased reflectivity of ice does not arise. In addition, having already a wet skin when it reaches the freezing level, the melting of the ice as it falls below the freezing level will have no effect on the power to reflect radar energy, except possibly that caused by a gradual small decrease in the size of the drop as the ice melts, and no bright band effect would be observed. The suggestion of wet hail therefore would appear to explain the radar facts. There are however arguments against this interpretation.

The interpretation requires the existence of wet hail in *all* cumulonimbus echoes which extend above the freezing level. Hail on the ground however is reported relatively infrequently in shower type precipitation, its distribution, except in the tropics, being much the same as that of thunderstorms (Sutcliffe 1940, p. 250) and exploration of cumulonimbus clouds by aircraft at heights above the freezing level have produced remarkably few reports of hail. Other aircraft reports have indicated moderate to severe icing inside the cumulonimbus weather echo well above the freezing level while at heights up to 3,500 ft. *above* the freezing level and in the echo the pilot has frequently reported the precipitation as rain. In

addition we have the report quoted above which implies the existence of water drops at temperatures as low as —39°F.

Findeisen and Schulz (1944) have shown that condensation on nuclei will occur almost exclusively as supercooled water droplets in cumulus clouds down to a temperature of —20°F and that even at temperatures of —40°F the number of ice crystals produced does not yet approach the number of water drops formed in the same space. Radar echoes however have been received on numerous occasions (14 out of a total of 50 in Fig. 1) when limited convection only was possible, with echo top temperatures of 10°F or higher. On many of these occasions the existence of an inversion or isothermal layer at or near the echo-top temperature made it certain that the cloud top was very close to the echo-top. On these occasions typical cumulonimbus type echoes were received — strong vertical columns of echo without bright band characteristics — and yet very few, if any, ice crystals can have been present in the cloud. Reports from tropical areas (*e.g.* Hunt 1949) indicate that moderate to heavy showers occur from clouds which do not extend to the freezing level and hence could not possibly have wet hail within them although there is little doubt that the radar echo from such clouds would be similar to those experienced from showers in this country.

Although the cumulonimbus echo *may* be interpreted in terms of wet hail there appear therefore to be strong arguments against this interpretation and the alternative suggestion is made that cumulonimbus echoes are principally (and perhaps often entirely) due to supercooled water drops of *raindrop size*. To explain the fact that such echoes frequently extend to temperatures below —10°F requires a theory to explain how liquid water drops of such a size can arrive at such a height. The work of Findeisen and Schulz quoted above shows that the existence of supercooled water at temperatures below —10°F cannot be called in question but its existence in raindrop size is not so readily explained. If all drops of raindrop size originate around an ice crystal as required by the Bergeron-Findeisen theory it is difficult to see why the drop should be liquid at temperatures below —10°F and the argument against hail has already been given above. The alternative would appear to be growth to raindrop size without the assistance of an ice crystal by condensation and coalescence in the liquid state. According to Findeisen (quoted by Simpson 1941) growth by condensation alone would take an impossibly long time but Simpson (1941) gives the argument in favour of growth to raindrop size by coalescence and condensation without the aid of ice crystals in strong vertical currents and it is suggested that the radar evidence may lend some support to this view.

Since to produce a radar echo on this equipment at a range of 11 — 20 mi, the drops must almost certainly be greater than 1·0 — 1·5 mm in diameter, and hence have a terminal velocity of the order of 5 m/sec, and since in many cumulonimbus clouds the vertical current must frequently exceed this value (Byers and Coons 1947, Byers and Braham 1948) drops of this size must frequently be carried upwards in the vertical current until they either have grown to a size which will permit them to fall down through the current or the current itself decreases to 5 m/sec or less.

The top of the echo *may* therefore be associated with a decrease of the vertical current speed to about 5 m/sec and we see why the echo-top temperatures decrease with increasing vertical speed within the cloud. In some cases the vertical current speed over a considerable interval of height may exceed the terminal velocity of the largest raindrop which can exist without disruption (about 10 m/sec) and in this case there would be no raindrops in this interval unless they were formed while travelling upwards in the vertical current or already formed below these heights. A continuous radar echo in these circumstances requires that growth to raindrop size should take place in the ascending current unless large falling hailstones provide the echo. Typical thunderstorm echoes (see Figs. 4 and 5) give no indication of any hollow core which would be necessary on any other explanation and the presence of large hailstones, while sometimes occurring, is not an invariable feature of the thunderstorm.

To the two differing types of bright band weather echoes we must therefore add, with strong vertical currents, possibly greater than 5 m/sec, a third entirely different echo, namely the cumulonimbus echo due to water drops alone.

We now have some evidence of two different methods of formation of rain :

(i) With small vertical current speeds less than about 1 m/sec such as occur

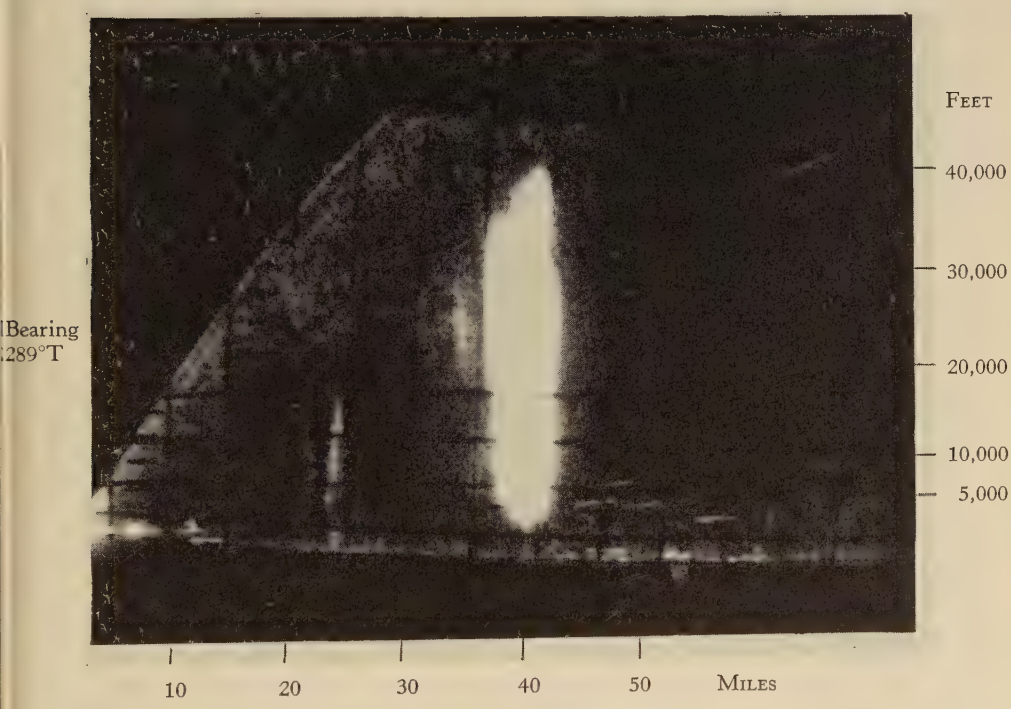

Figure 4. 14 July 1947, 14.22 G.M.T. This is a typical thunderstorm echo. Note the similarity to ordinary cumulonimbus echoes (Fig. 3) although on a much larger scale.

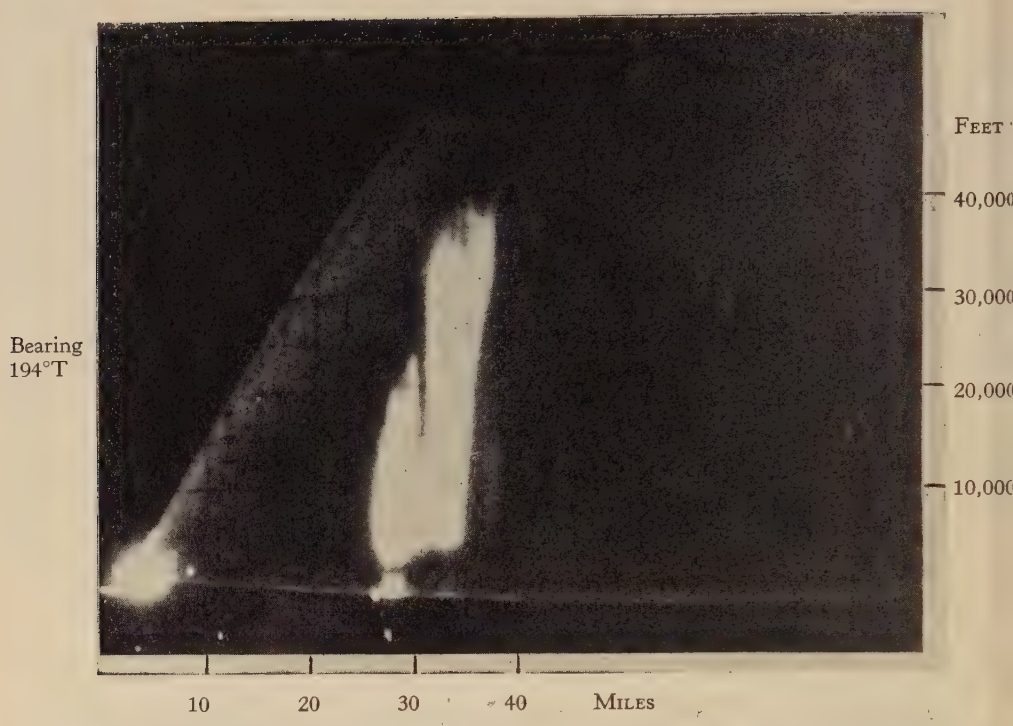

Figure 5. 30 July 1948, 15.44 G.M.T. This is also a thunderstorm echo : note the tendency for the echo column (and hence probably the vertical currents) to diverge near the top. The nearer column of echo was very probably a strongly developing new cumulonimbus at the edge of the main thunderstorm mass. Development of this character has often been noticed by the pilots investigating cumulonimbus clouds.

in layer and frontal clouds, the formation of rain from the melting of large ice crystals or snowflakes formed from an aggregate of crystals at or about the freezing level and leading to the bright band radar phenomena.

(ii) The formation of rain without the assistance of ice crystals in strong vertical currents by growth and coalescence in the liquid state.

Examples have been observed when a bright band has been seen close to a shower type of echo and this can be explained if we regard the bright band as evidence of a reduction in the vertical current as may occur, for instance, in a degenerate cumulonimbus cloud.

6. The echo intensities from thunderstorms

An examination of Table III shows that not only is the temperature at the top of a thunderstorm echo (when allowance is made for range) probably much lower than for other echoes, except in the one exceptional case already referred to (4 October 1948), but also, with this same exception, that the maximum signal intensity is probably much greater than that from most other weather echoes. In Table IIIB there are only three occasions when thunderstorms were probably not

present which give echo intensities comparable with those from thunderstorms. Two of these occasions (for 25 August 1948 and 4 May 1948) were assocated with particularly active fronts while the third occasion on 22 July 1948 was viewed at a range of 48 mi to the NW. On this latter occasion, although no thunder was reported from the Midlands and South of the country, thunder was reported in South Yorkshire and Lancashire and this echo may therefore have come from an isolated storm of short duration.

The conclusion seems justified that not only does the weather echo from a thunderstorm usually extend to a lower temperature than other weather echoes but also that the intensity of the received echo is usually much greater. In exceptional conditions a very strong echo may be received from non-thunderstorm precipitation and very exceptionally (4 October 1948) a thunderstorm may give an echo indistinguishable from that from many other echoes not associated with thunderstorm conditions.

If we accept the theoretical deduction that the echo intensity is proportional to ΣND^6 (Ryde 1946), where N is the number of drops of diameter D per unit volume, we see that the increased echo intensity can be explained either by a great increase in the number of drops per unit volume or by a much smaller proportionate increase in the diameter of the drops or by a combination of the two effects. It seems likely that the great increase in signal intensity in changing from an ordinary shower to a thunderstorm is due to an increase in the size of the drops since, with the stronger vertical currents, the life of a waterdrop within the cloud will be appreciably longer and give greater opportunity for the growth of the drop.

Some light is thrown, therefore, on the physical properties which distinguish a thunderstorm from other cumulonimbus clouds, namely that for a thunderstorm the vertical currents and raindrop size are usually greater than in an ordinary cumulonimbus. This may lend support to the " breaking drop " theory of separation of electrical charge within a thunderstorm.

It may also be significant that the temperature at the top of the thunderstorm echo is usually below about —40°F and therefore in the region where a sudden increase in the number of ice particles is likely to take place (Findeisen and Schulz 1944). This suggests that a contributory factor to the generation of a thunderstorm may be the introduction of a number of supercooled raindrops into a volume containing large numbers of ice crystals. Infrequently, large numbers of ice crystals may exist at higher temperatures than —40°F — as the remains, for example, of previous frontal activity — and the introduction of supercooled raindrops into such a volume may explain the exceptional thunderstorms with a relatively high exho top temperature and relatively low echo intensity.

Acknowledgments

I am indebted to the Director of the Meteorological Office for permission to submit this paper.

My thanks are also due to Mr. A. C. Best of the Meteorological Office for valuable criticism and advice.

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Discussion on radar echoes

The February 1950 meeting of the Society took the form of a discussion on "Radar Echoes," a Vice President, Dr. R. C. Sutcliffe, O.B.E., being in the chair.

The openers of the discussion were Mr. J. E. N. Hooper, of Telecommunications Research Establishment, Malvern, who read the paper "The bright band — a phenomenon associated with radar echoes from falling rain" (p. 125 of this volume) by himself and Mr. A. A. Kippax, and Mr. R. F. Jones, of the Meteorological Office, who read his paper "The temperatures at the tops of radar echoes associated with various cloud systems" (p. 312 above).

Discussion

Mr. B. J. Mason : Mr. Jones in interpreting the radar echoes from precipitating clouds has made some very provocative suggestions as to the physical processes involved, many of which I find difficult to accept. I will confine my remarks to two points only which he has raised.

I cannot agree that the occasional absence of the bright band from cumuliform clouds necessarily indicates the absence of the ice phase. It appears more probable that in such a case the ice particles are covered with a film of liquid water, — reflection of the electromagnetic wave will occur at this film, the element appearing as far as the radar echo is concerned as a water drop. In fact, the bright band is observed in the later stages of development of a cumulus, although it is generally more diffuse than from layer clouds. This lack of definition may be explained by the fact that

the stronger updrafts in cumulus cause the coagulation elements to be distributed over a wider region about the freezing level. In order to defend his thesis that ice is sometimes absent from cumulus clouds in these latitudes, Mr. Jones appeals to observations of rain from tropical cumuli whose tops do not reach the freezing level, and sees no reason to distinguish between the two cases. The evidence for heavy precipitation from individual cumuli which could not have been seeded at some stage in their history is indeed scanty ; this is not to say, however, that such a process does not take place occasionally, but it seems to be the exception rather than the rule. Surely, one very significant difference between tropical and temperate cumuli is the much higher water contents of the former, which would lead to a higher growth rate of drops by a collision process.

Mr. R. Wexler : The reflectivity of the melting snowflake may be shown to be proportional to K^2/V where V is the fall velocity and $K = \frac{\epsilon^2-1}{\epsilon^2+2}$ and ϵ is the dielectric constant. The factor K^2 is 0·94 for water and 0·19 for ice. Ryde assumed for the melting snowflake that $K = m_i K_i + m_w K_w$ where the subscripts i and w refer to ice and water and m_i and m_w are the respective percentages of ice and water. This assumption is valid for a heterogeneous mixture of ice and water. If it is also assumed that $V = V_i m_i + V_w m_w$ then no bright band occurs since the increase in dielectric constant is cancelled by the simultaneous increase in fall velocity. Since the water of the melting snowflake is mostly on the outside, it is likely that the dielectric constant of the mixture is more nearly that of the water and that Ryde's assumption underestimates the value for K. The more rapid increase in dielectric constant and a very gradual increase of the fall velocity would result in the bright band. Admittedly, however, little is known concerning the fall velocity of a melting snowflake.

It is difficult to see how Messrs. Hooper and Kippax were able to separate the snowflake into its component parts. Beyond a diameter of 0·1 mm snow crystals tend to grow dendritically owing to their motion. This sprout growth may well result in a complex snowflake mass which may appear to be made up of a coalescence of ice crystals. Because of the small concentration, of the order of 1 per 100 cm^3, it is difficult to visualize chance collisions of many ice crystals into one snowflake. For equal growth conditions the thicknesses of the sprouts are equal, so that collections of equal numbers of sprouts of similar appearance may well result in equal masses.

Mr. I. C. Browne : I should like to summarise observations, made at Cambridge with the equipment used by Hooper and Kippax, in so far as these refer to the bright band.

The bright band is nearly always present. It is most prominent in warm-front rain and least prominent in the initial stages of a thunderstorm, while it may be absent in extremely slight drizzle (two lantern slides illustrated the weak bright band at the beginning of thunderstorms). The width of the band varies with the type of rain ; it is measured by comparing the width of the bright band echo with the width of the echo from an object such as a factory chimney — this gets over the difficulty due to pulse broadening in the receiver. The band is widest

in thunderstorms and the width probably indicates turbulence of spread of snow-crystal size.

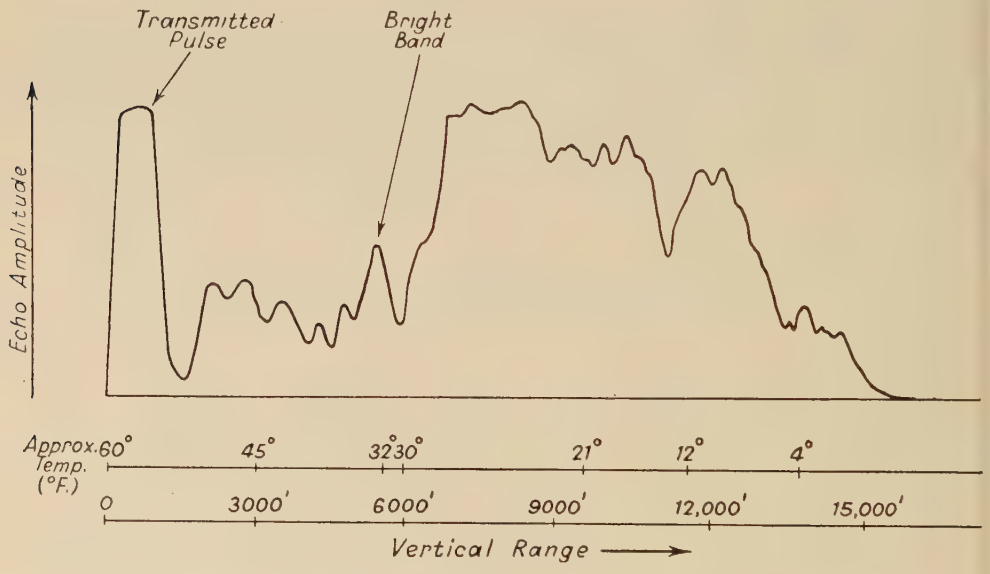
An interesting case of a bright band occurred when the freezing level and bright band were at 3,000 ft while snow flakes were reaching the ground — the ground temperature was slightly below freezing and nowhere were temperatures above freezing. This indicates that the bright line is formed as soon as water forms on the snowflake, but the echo could not be studied much below 3,000 ft owing to receiver paralysis.

The thunderstorm observations imply that Mr. Jones is incorrect in stating that ice is not present initially in thunderstorms, but they do not prove that the ice is essential to rain production. His failure to observe the bright line may be due to

(1) relative insensitivity of intensity-modulated displays as compared to deflection-modulated displays.

(2) drop in resolving power due to angular spread of beam. For example, a $1\frac{1}{2}°$ beam is 1,500 ft wide at a range of 10 mi (Mr. Jones's minimum working range) and this is several times wider than the usual bright band.

The accompanying figure shows the weak bright band in the very early stages of a thunderstorm. The photograph from which the drawing was made was taken within a few minutes of the first appearance of an echo from the cloud and within less than 30 seconds of the first rain reaching the ground. Although numerous parts of the echo are stronger than the bright band, the latter persists in the same place through all the photographs, strengthening toward the later stages of the storm. The temperatures shown are taken from the *Daily Weather Report*.



Radar echo from thunderstorm in initial stages. (Rain has begun to fall).

Mr. J. L. Galloway : The photograph of Mr. Hooper's Radar A scope during precipitation is quite unlike any radar cloud photograph I have previously seen and I should be glad if Mr. Hooper would amplify the information given in his paper as to his method of arranging his apparatus. Mr. Jones, with 10-cm radar, obtained a bright band effect that was identifiable, but with 3-cm radar exposed vertically Mr. Hooper obtained a degree of resolution that made practical use of the effect possible. I think that its use would be invaluable at airports in affording the Controller up-to-the-minute information as to the height of the freezing level in heavy weather. Perhaps some indication could also be obtained as to the rate of ice accretion likely to be experienced. I wonder if existing radar installations could be used or easily adapted for the purpose ?

It would be interesting to know what is the minimum height of the freezing level for detection and whether the amount of separation of the bright band and freezing level varied with height above the surface. Presumably the method would fail in snowfall with surface temperature above 32° F.

Mr. F. H. Ludlam : Mr. Wexler and I have recently been considering quantitatively the development of precipitation within cumulonimbus clouds, assuming that it is initiated with the appearance of the ice phase. It appears not only that much of the hail may be wet with a temperature close to 0°C, and thus very readily melted below the freezing level, but even that the bulk of the precipitation in the cloud may be composed of liquid water rather than ice. The observation of rain within cumulonimbus at sub-freezing temperatures is therefore not incompatible with the view that its development is due to the appearance of ice crystals in the cloud.

Mr. E. Gold : I admire the way in which Messrs. Hooper and Jones have presented the results of their investigations. I think there can be no doubt about the soundness of the explanation of the " bright band " which has been put forward. I suggest that in the bright-band level the mixture was partially melted snowflakes and not one of completely melted flakes and of unmelted flakes : and in these circumstances the change in reflectivity would not be counterbalanced by the change in the rate of fall of the ' wet ' snowflakes. Though snow is essential to the explanation (and snow does actually occur) it seems difficult in view of the results described by Mr. Jones to see how snow could even be formed. Is the absence, from Fig. 3 of the paper by Hooper and Kippax, of any flakes with 7, 9 or 11 crystals an accidental omission or the actual result of the observations ? It seems confusing. I consider that the results of the investigations show what a powerful means for examining such theories as that which required the presence of ice crystals for the formation of natural rain, is now available to meteorologists and I feel that all meteorologists will be grateful for it.

Dr. Fournier D'Albe : It strikes me that one point concerning precipitation echoes, of importance to the physics of the problem, has not yet been mentioned : that is, the height and temperature at which the echoes first appear. Mr. Jones has analysed for us the temperatures at the tops of echoes from various types of cloud, including convection clouds, but since these appear to have been measured

at the time of maximum development of the clouds concerned they will merely tell us something about the inversion or isothermal layer which limits the upward growth of the clouds.

A knowledge of the temperature at which precipitation echoes first appear would however give us valuable insight into the process involved and in particular would indicate whether or not ice crystals may be present ; though to be certain on this latter point it would be necessary to make simultaneous visual observations of the height of the cloud top.

Mr. D. L. Champion : There appears to be some confusion about the meaning of the terms *ice crystals, snow crystals* and *snow flakes*. G. Seligman, in his excellent book on *Snow Structure and Ski Fields* (1935) gives some considerable data on the subject, and I feel that the term *ice crystals* should be applied to the basic hexagonal plates. The next stage of development (with the additional six dendrites) are conveniently termed *snow crystals* ; and interlocked agglomerates of the latter should be called *snow flakes*.

The individual six-rayed snow particles seem to me to be far too small and crystalline in appearance to be referred to as flakes, whereas an accumulation of these does tend to take a more laminar shape akin to a flake.

Dr. T. W. Wormell : Mr. Hooper on the one hand, and both Mr. Jones and the film shown by Mr. Wexler on the other hand, have illustrated the potentialities of radar as a method of investigating the structure and development of precipitation clouds. The methods of presentation described by Mr. Jones make possible a rapid survey of conditions over a wide area. At the risk of stressing the obvious, may I recall the necessity of remembering, when examining the structure of cloud in a vertical plane, as shown on some of Mr. Jones's slides and in the film, that the vertical scale of the picture is greatly exaggerated relative to the horizontal scale. I make this remark because I have not infrequently heard such pictures of echoes from showers and thunderstorms described as having the appearance of vertical fingers. In fact, the volume in a cumulonimbus which gives a radar echo normally has vertical dimensions of the same order as, or even less than, its horizontal dimensions.

The method used by Messrs. Hooper and Kippax of " looking " vertically upward with the radar set gives much more detailed information concerning local clouds. We are now fortunate, at Cambridge, in having the equipment developed at T.R.E., and described by Mr. Hooper, and are attempting to exploit its possibilities for the detailed examination of cloud structure and its rapid variation in time and space. Mr. Browne has shown some examples of the echo structure from overhead thunderstorms. Noteworthy features of such photographs are first, the almost invariable appearance of an intensity peak at about the freezing level, although its strength may be exceeded by the echo from other heights, and secondly, on many occasions, the great strength of the echo from an extended range of heights well above the freezing level. Whether one calls the peak at the freezing level a " bright line," when its strength is exceeded by the echo intensity from other heights, is largely a matter of language, but the regular presence of the peak would

seem to constitute clear proof that there is normally in a thundercloud something to melt at the freezing level.

Mr. J. E. N. Hooper (*in reply*) : I am indebted to Mr. Mason for pointing out what was, I think, only an error in nomenclature when I referred to coagulation of crystals ; I should have referred to the coagulation of single dendritic flakes to form large composite flakes. I should like to reaffirm that it was quite possible to identify up to 12 such individual flakes in a large flake ; it so happened, however, that of the comparatively few flakes so examined none was composed of 7, 9 or 11 single flakes.

Fig. 1 shows that above the freezing level the echo intensity increases slightly with decreasing height and Mr. Wexler infers this to be due to growth of the flakes. This may be true but it should not be overlooked that echo intensity will apparently increase in this way due to the range factor alone. Mr. Wexler stated that the echo intensity from below the bright band is some five times greater than that from above. Fig. 2 shows that on that occasion they were about equal and Mr. Kippax has pointed out that the latter intensity is very variable and apparently difficult to correlate with the former, and for this reason we have made no detailed study of its relative magnitude.

Mr. Wexler has quoted expression (20) from Ryde's paper (Ryde 1946) and has pointed out that if a corresponding expression relating to velocities is combined with it then no bright band can be expected. If, however, the flakes begin to melt at the extremities of the arms then it is not unreasonable to suppose that a considerable amount of the flake will have turned to water before the various droplets coalesce and the velocity changes appreciably.

I would expect that a conventional marine radar set would be unlikely to detect a bright band lying near to the surface since angular discrimination between it and ground or sea clutter would be very difficult.

I should like to emphasise that the observation of the bright band within echoes from rain on an intensity modulated display (*e.g.* height-range, height-elevation) is very difficult unless the gain is at a low setting. This type of display effectively limits signals at a comparatively low level so that gradations in echo intensity about this level are indistinguishable.

Mr. Gold's observation to the effect that in radar the meteorologist has acquired a new tool was particularly apt and if the papers and discussion this evening have done nothing more than emphasise this point, the meeting has not been in vain.

Mr. R. F. Jones (*in reply*) : I agree with Mr. Mason that a possible explanation of the absence of bright band in cumulonimbus echoes may be the existence of a film of water on the ice crystals and this point was made in my paper. However, I endeavoured to give the arguments against this interpretation and I still consider that these arguments are valid. There seems little doubt that rain does fall from some tropical cumuli which fail to reach the freezing level and that, in these circumstances, ice crystals can have played no part in the production of the rain. I see no reason why there should be one type of shower rain in the tropics and another in the temperate zones. Mr. Mason quotes the much higher water content

of tropical cumuli as an adequate reason, but the amount of free water made available by ascent of air in a tropical cumulus which fails to reach the freezing level may not be much greater than the amount of water set free in a temperate cumulus. (Rough calculations on a tephigram show 10·5 g/kg set free in tropical air with a surface dew point of 76° F and cloud base at 950 mb ascending to 550 mb (temp. 40° F) and about 7·1 g/kg for a temperate cumulus with dew point 57° F and cloud base 950 mb of the same vertical extent (temp. at top 12° F), while ascent to a temperature of —20° F sets free about 9 g/kg).

In reply to Messrs. Browne and Wormell, I have suggested that the presence of ice may be an important factor in the electrical activity of thunderstorms, and the appearance of ice in the later stages of thunderstorm growth is not therefore surprising — the presence of a bright band in such situations has also been observed with our equipment. It would be interesting to know whether an intensity peak has also been observed in newly developing cumulonimbus clouds which did not become thunderstorms. It is agreed that the height-range tube display is not ideal for viewing the bright band (a point also made by Mr. Hooper) but it is possible, by reducing the gain of the receiver, to see where the maximum signal intensity occurs in the echo, although small intensity peaks, such as Dr. Wormell describes, the strength of which is exceeded at greater heights, would be difficult to observe. It has been a regular practice in viewing shower echoes to reduce the gain gradually until the echo is extinguished without seeing any indication of a horizontal bright line, except when there has been good reason to believe the shower cloud to be in the decaying stage.

I am glad that Mr. Ludlam can support on theoretical grounds my suggestion that the radar echo from a cumulonimbus is principally from liquid water drops, but he does not answer all the objections put forward in my paper to the ice crystal as originator of the precipitation.

With reference to Mr. Gold's remarks, my suggestion that snow is largely absent applied only to cumulonimbus clouds in which the bright band is also considered to be absent, the existence of snow and the bright band, and its explanation, in layer-type clouds is not questioned and is indeed discussed at some length in my paper.

In reply to Dr. Fournier D'Albe, it is true that the maximum top of a cumulonimbus echo is probably very closely related to the height of the stable layer which limits further development, but the same is certainly not true of the layer-type clouds. To determine the temperature at which radar echoes first appear, together with observation of the cloud top at that time, would certainly be most valuable but the observational difficulties are very great.

Relation between the standard deviations of temperature at constant pressure and at constant height

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Meteorological Office

(Manuscript received 4 May 1949, revised 23 August 1949)

1. Introduction

In order to give information on the frequency of upper air temperatures for aircraft operation and other purposes, it is necessary to draw world maps (a) of mean temperature, and (b) of the standard deviation of temperature. Since aircraft commonly fly on a pressure surface, the maps should also be prepared for pressure surfaces. As most data at present available regarding standard deviation of temperature are related to actual heights, the first thing is to establish the relationship between the standard deviation at constant pressure and that at constant height, and this has been done in the following note. The formula, which is given as Eq. (2) below, was first suggested in a manuscript note by Miss N. Carruthers, but she has since agreed that the reasoning leading to it was inadequate. The following proof has been derived independently.

2. The formal relationship

We consider a set of pairs of observations of temperature T_p at a fixed pressure p, and temperature T_h at a fixed height h, where p and h refer to the same average level. The mean temperature at this level will be denoted by T, and we write for the individual occasion,

$$T_p = T + \delta T_p, \qquad T_h = T + \delta T_h \qquad . \qquad . \qquad . \qquad (1)$$

On the same occasion, h occurs at a pressure which may be denoted $p + \delta p$, so that if $\delta p/p$ is sufficiently small, then

$$T_p - T_h = -\frac{\partial T_p}{\partial p}\,\delta p,$$

whence from (1),

$$\delta T_p = \delta T_h - \frac{\partial T_p}{\partial p}\,\delta p$$

Squaring,

$$(\delta T_p)^2 = (\delta T_h)^2 - 2\,\delta T_h \cdot \frac{\partial T_p}{\partial p}\,\delta p + \left(\frac{\partial T_p}{\partial p}\right)^2 (\delta p)^2$$

The standard deviations of temperature at constant pressure and at constant height are given respectively by

$${}_p\sigma_T{}^2 = \overline{(\delta T_p)^2}, \qquad {}_h\sigma_T{}^2 = \overline{(\delta T_h)^2},$$

a bar denoting the mean of a set of observations, so that by taking means of each term in the preceding equation we obtain

$$ {}_p\sigma_T^2 = {}_h\sigma_T^2 - 2\,\overline{\delta T_h \frac{\partial T_p}{\partial p}\,\delta p} + \overline{\left(\frac{\partial T_p}{\partial p}\right)^2 (\delta p)^2} $$

Now apart from the lowest layers of the atmosphere influenced by frequent surface inversions, a little consideration shows that $\partial T_p/\partial p$ can be replaced by its mean value $\partial T/\partial p$, with neglect only of small terms of third and higher orders. Consequently the equation becomes

$$ {}_p\sigma_T^2 = {}_h\sigma_T^2 - 2\,\frac{\partial T}{\partial p}\,\overline{\delta T_h\,\delta p} + \left(\frac{\partial T}{\partial p}\right)^2 \overline{(\delta p)^2} $$

Also

$$ \overline{\delta T_h\,\delta p} = {}_h r_{pT} \cdot {}_h\sigma_p \cdot {}_h\sigma_T $$

and

$$ {}_h\sigma_p^2 = \overline{(\delta p)^2} $$

where ${}_h r_{pT}$ is the correlation coefficient between pressure and temperature at height h (which is generally positive in the free air), and ${}_h\sigma_p$ is the standard deviation of pressure at height h. Therefore

$$ {}_p\sigma_T^2 = {}_h\sigma_T^2 - 2\,\frac{\partial T}{\partial p} \cdot {}_h r_{pT} \cdot {}_h\sigma_p \cdot {}_h\sigma_T + \left(\frac{\partial T}{\partial p}\right)^2 \cdot {}_h\sigma_p^2 \quad . \quad . \quad (2) $$

which is the required relationship expressing the standard deviation of temperature at constant pressure in terms of functions referring to a fixed height. With a transformation of the factor $\partial T/\partial p$, it may also be written

$$ {}_p\sigma_T^2 = {}_h\sigma_T^2 + \frac{2\lambda}{g\rho} \cdot {}_h r_{pT} \cdot {}_h\sigma_p \cdot {}_h\sigma_T + \left(\frac{\lambda}{g\rho}\right)^2 \cdot {}_h\sigma_p^2 \quad . \quad . \quad (3) $$

where λ is the mean vertical temperature gradient and ρ the mean density at height h.

3. Comparison of the two standard deviations

The value of $\partial T/\partial p$ in the I.C.A.N. atmosphere is instanced by the following figures :—

Pressure (mb)	700	500	300
$\frac{\partial T}{\partial p}$ (°C/mb)	·073	·097	·142

From the figures given below it is seen that since ${}_h\sigma_p$ is comparable with ${}_h\sigma_T$ and the coefficient $(\partial T/\partial p)^2$ is relatively small, it follows that in the troposphere

$$ {}_p\sigma_T < {}_h\sigma_T $$

In the stratosphere, where $\partial T/\partial p$ is zero, the two standard deviations are equal.

As an illustration, the following figures have been evaluated for Larkhill, England from midnight ascents.

Period 1947	p mb	$_h\sigma_T$ °C	$_h\sigma_p$ mb	$_h r_{pT}$	$\partial T/\partial p$ °C/mb	$_p\sigma_T$ °C	
						Obs.	*Calc.*
January	500	6·1	9·1	0·87	0·10	5·4	5·3
	300	3·1	8·0	0·49	0·12	2·8	2·8
July	500	3·9	4·1	0·76	0·10	3·4	3·6
	300	3·4	5·9	0·58	0·16	3·0	3·0

These show very satisfactory agreement between the observed values of the standard deviation of temperature at constant pressure, and those calculated from equation (2).

Some data are also available for Agra (Chiplonkar 1936) from which the following values have been derived.

Period 1926—31	h km	p mb	$_h\sigma_T$ °C	$_h\sigma_p$ mb	$_h r_{pT}$	$\partial T/\partial p$ °C/mb	$_p\sigma_T$ °C
							(Calc.)
December—	4	617	3·6	3·7	0·76	0·08	3·4
February	6	475	3·3	4·2	0·79	0·11	3·0
	8	359	3·8	4·2	0·87	0·14	3·3
	10	267	4·7	3·9	0·55	0·15	4·4
September—	4	623	2·9	1·5	0·26	0·07	2·9
October	6	483	4·4	2·0	0·57	0·09	4·3
	8	370	3·8	2·6	0·84	0·14	3·5
	10	279	3·6	3·2	0·88	0·19	3·1

Observed values of $_p\sigma_T$ are not available in this case. From these two tables, it is seen that the largest difference between the two standard deviations of temperature, $_h\sigma_T$ and $_p\sigma_T$, is 0·7°C, the former being always the greater, apart from one case of equality.

Since the standard deviations and correlation coefficients do not vary greatly with height within the troposphere, the largest difference between the two standard deviations of temperature may be expected to occur in the upper troposphere on account of the increase with height of the factor $\partial T/\partial p$ or $-\lambda/g\rho$ (Eqs. (2) and (3)). A rough calculation based on the figures for Agra but on a value of $\partial T/\partial p$ at 50,000 ft about double that at 10 km suggests that the greatest difference between $_p\sigma_T$ and $_h\sigma_T$ is probably about 1°C and occurs below the equatorial tropopause. A similar difference appears likely in the upper troposphere in middle latitudes when seasonal values are considered.

4. Conclusion

Summing up, we see that the excess of the standard deviation of temperature at constant height over that at constant pressure is positive and in general tends to increase with height up to the neighbourhood of the tropopause but becomes zero in the stratosphere. The available figures indicate that for seasonal values the difference may amount to about 1°C in the upper troposphere.

Acknowledgment

Grateful acknowledgment is made to Mr. C. S. Durst, for proposing the problem, to Mr. J. S. Sawyer, and to Mr. A. G. Matthewman, for helpful suggestions, and to the Director, Meteorological Office, London for permission to publish the paper.

Reference

Chiplonkar, M. W. 1936 *Beitr. Phys. frei Atmos.*, Leipzig, **23**, p. 238.

551.513 : 551.555.1

Some comments on the trade-wind cell

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Communicated by R. C. Sutcliffe 4 April 1950

In a previous article (Riehl and Yeh 1950) the mean meridional circulation at the surface was computed for January and July between latitudes 45° N. and 45° S. This computation reaffirms the classical description of the trade wind cell. But it also indicates the need for a revised intepretation of certain features.

According to previous explanations, the descent of air in the trade wind belt is brought about by radiational heat losses in the upper troposphere. The poleward moving antitrades become relatively heavy and sink. This has been regarded as a weak link in the description of the general circulation in low latitudes for some time. Recent observations throw further doubt on its validity. We do not observe a slow general drift of a broad current aloft which would permit the gradual cooling mentioned. On the contrary, strong eddy motion prevails at high levels (Riehl 1950). Yet the subtropical divergence is a semi-permanent feature over wide areas, as seen from statistics on cloudiness and precipitation. It is therefore more reasonable to connect the subtropical divergence with conditions near the surface, where we encounter the only quasi-steady link of the general circulation in the troposphere.

As we follow the trajectories of north-east and south-east trade from the subtropical ridgeline equatorward, we observe that the absolute vorticity (ζ_a) of the

air everywhere is nearly equal to the vorticity of the earth itself (f). Individual particles thus must lose absolute vorticity during their traverse of the trade wind belt. We shall apply the vorticity theorem

$$\frac{d\zeta_a}{dt} = -\zeta_a \operatorname{div}_2 V \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\frac{d}{dt}$ is the substantial derivative. The effects of the horizontal solenoids and of skin friction have been omitted in this form of the vorticity theorem. It can be shown from observations that the solenoid term has a magnitude of only 10 per cent or less of that of the divergence term. Skin friction does not affect the vorticity if there is no lateral shear (provided the coefficient of skin friction is constant). Since the shear in the trade wind belt is small, this omission may not be serious for a first approximation.

Considering the trade as a steady circulation system and setting

$$\frac{d}{dt} = u \frac{\partial}{r \cos\phi \partial\lambda} + v \frac{\partial}{r\partial\phi}$$

for steady motion along the surface, Eq. (1) becomes

$$u \frac{\partial\zeta_a}{r\cos\phi\partial\lambda} + v \frac{\partial\zeta_a}{r\partial\phi} = -\zeta_a \left(\frac{\partial u}{r\cos\phi\partial\lambda} + \frac{\partial v}{r\partial\phi} - \frac{v}{r}\tan\phi \right) \quad . \quad . \quad (2)$$

Here, ϕ is the latitude, λ the longitude, r the radius of the earth, and u and v are the velocity components along λ and ϕ respectively. The term $\frac{v}{r}\tan\phi$ has the order of magnitude of only 10^{-7} sec^{-1} in the tropics compared with 10^{-6} sec^{-1} for the other two terms in the expression for divergence. It will therefore be neglected. Rearranging,

$$\frac{\partial}{\cos\phi\partial\lambda}(u\,\zeta_a) + \frac{\partial}{\partial\phi}(v\,\zeta_a) = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Integrating around a latitude circle,

$$\frac{\partial}{\partial\phi}\int_0^{2\pi} v\,\zeta_a\,d\lambda = 0,$$

and integrating with respect to latitude,

$$\int_0^{2\pi} v\,\zeta_a\,d\lambda = \text{constant.} \quad . \quad . \quad . \quad (4)$$

If we set $\zeta_a = f + \zeta_r$, where ζ_r is the relative vorticity,

$$\int_0^{2\pi} f v d\lambda + \int_0^{2\pi} \zeta_r\, v d\lambda = \text{constant.} \quad . \quad . \quad . \quad . \quad (5)$$

Since $\zeta_r << f$ excepting the inner equatorial zone, the second term of Eq. (5) should be small compared to the first. As a numerical example, the average values of the two terms at latitude 17·5° N. in January are 8·2 and 0·33 $\times$ 10^{-5} m sec^{-2} per unit length of the latitude circle. The second term amounts to only 4 per

cent of the first and we can write with good approximation,

$$f\bar{v} = \text{constant}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\bar{v}$ now is the mean meridional flow shown in Fig. 1.

The basic proposition contained in the foregoing development is as follows: the trades, moving equatorward, continually import high absolute vorticity from high toward low latitudes. *This vorticity is removed by lateral divergence at the same rate at which it is imported. If the trades possess a high degree of steadiness from other considerations (cf.* Riehl 1950), *it follows that divergence must be present within the trades.* It is of interest to note in view of the foregoing that the mechanism for the observed downstream increase in the height of the trade wind inversion is in need of re-examination*.

For verification of Eq. (6), Fig. 1 shows the profiles of v and of divergence for the northern hemisphere winter together with the theoretical distribution of $\bar{v}$ demanded by Eq. (6). Agreement is excellent in the latitude belt with divergence.

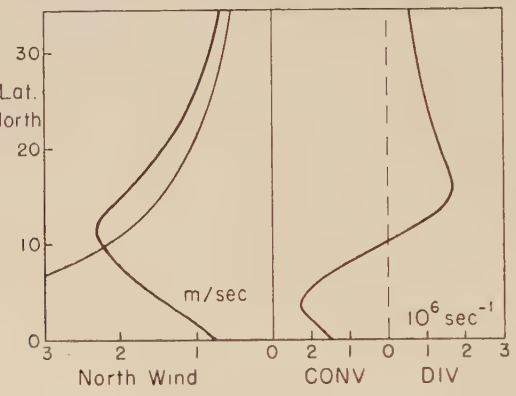

Figure 1. Left: Profile of net meridional flow in January (m sec^{-1}) against sine of latitude (heavy line) and theoretical distribution as computed from Eq. (6) (thin line).
Right: Corresponding distribution of actual divergence (10^6 sec^{-1}).

The value of the constant in Eq. (6) is $0{\cdot}5 \times 10^{-2}$ cm sec^{-2} in this profile. In northern and southern summer (*cf.* Riehl and Yeh 1950), Eq. (6) probably also is satisfied. But the zones of divergence within the trades are too narrow to permit a definite conclusion. The southern hemisphere winter is disappointing. Values of $\bar{v}$ increase more rapidly with decreasing latitude than prescribed by Eq. (6). A detailed check showed that the Indian Ocean is responsible for this. If a separate calculation is made for Atlantic and Pacific Oceans alone, Eq. (6) verifies between latitudes 12° and 27° S.

References

Riehl, H. and Yeh, T. C. 1950 *Quart. J. R. Met. Soc.*, London, **76**, p. 182.
Riehl, H. 1950 *Tellus*, Stockholm, **2** (1), p. 1.

* Without discussion, we refer to the mechanism proposed in "Vertical motion and exchange of heat and water between the air and the sea in the region of the Trades," Woods Hole Oceanographic Institution, October 1946, mimeographed.

551.513 : 551.552

North-south profiles of the average zonal wind component at the surface

By CHARLES L. JORDAN
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(Manuscript received 3 April 1950)

The observations recently used by Riehl and Yeh (1950) for a computation of the net meridional circulation at the surface can be employed also to determine the average zonal flow component as a function of latitude. In the *Atlas of Climatic Charts of the Oceans* (1938) resultant wind vectors are given for each 5° square. From these resultants, the zonal and meridional components in each square were obtained between latitudes 45° N. and 45° S. for January and July. The average value of each component for each latitude belt was then determined.

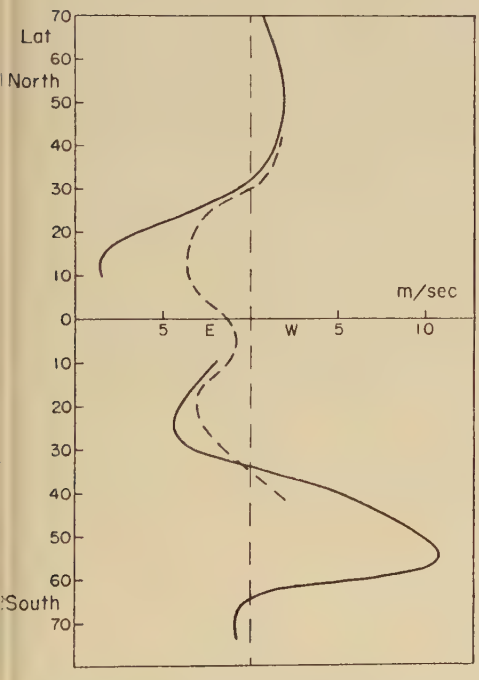

Figure 1. Latitudinal profiles of mean zonal wind (m sec^{-1}, broken line) and of geostrophic wind (solid lines) at the surface in January.

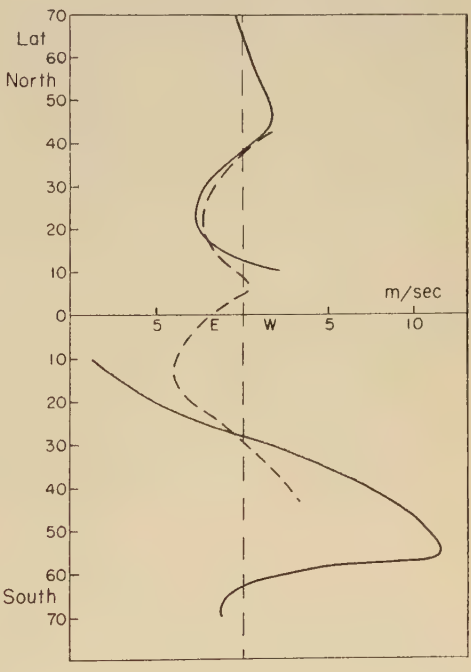

Figure 2. Same as Fig. 1 for July.

Figs. 1 and 2 show the latitudinal distribution of the mean zonal wind component, together with the geostrophic wind profiles between latitudes 10° and 70° as calculated from sea-level pressures given by Scherhag (1948). The present computation is of most value in the latitude belt 20° N. to 20° S. where the pressure distribution gives only a poor indication of the wind field. In this belt, the percentage of the earth's surface covered by ocean varies from 70 to 80. Therefore the profiles should be quite representative for the whole area. It is of particular interest that two zones of maximum east wind are evident both in January and July. The stronger maximum lies in the winter hemisphere. Between these belts we encounter lighter easterlies near the equator (doldrums). Strong westerlies in the Indian Ocean in July account for the small belt of resultant west wind between latitudes 5° and 10° N. in that month.

Outside the tropics, the agreement between actual and geostrophic wind computations is generally good. The positions of zero zonal wind in the subtropical ridge fall within 2° latitude in each of the four cases. In the belts of westerlies we note a sizable reduction of actual compared with geostrophic wind in the southern hemisphere, but not in the northern. This latter discrepancy may be ascribed to the fact that the ocean coverage is only 45 to 50 per cent along the northern hemisphere belt of westerlies. Inclusion of winds over the land areas would undoubtedly reduce the resultant speeds.

References

Riehl, H. and Yeh, T. C. 1950 *Quart. J. R. Met. Soc.*, London, **76**, p. 182.

Scherhag, R. 1948 *Neue Methoden der Wetteranalyse und Wetterprognose*, Berlin (Springer-Verlag), p. 57.

U.S. Weather Bureau 1938 *Atlas of Climatic Charts of the Oceans*, Washington, D.C. (U.S. Government Printing Office).

551.554

The ratio between observed velocities of the wind at 50 feet and 2000 feet over the North Atlantic Ocean

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(Manuscript received 27 August 1949)

Summary

The ratio of the wind at 50 ft to the wind at 2000 ft has been calculated for a number of cases from ocean weather ship observations over the North Atlantic and mean values classified according to the lapse rate and the velocity of the wind at 2000 ft. Curves have been drawn showing the variation of the ratio v_{50}/v_{2000} with lapse rate and v_{2000} respectively.

1. The ratio between surface and geostrophic winds over the sea

In a recent article in the *Meteorological Magazine* C. S. Durst (1949) gives a ratio of 0·80 and 0·66 for v_s/v_g for onshore winds at Gorleston in winter and summer where v_s and v_g are the surface and geostrophic winds respectively.

Shaw (1930) gives the following mean ratios for v_s/v_g for well exposed shore stations.

TABLE I. v_s/v_g FOR WELL EXPOSED LAND STATIONS

Station	Exposure	Mean v_s/v_g
Holyhead	Flat island	0·59
Spurn Head	Spit of sand	0·65
Yarmouth	Spit of sand	0·51
Scilly	Island	0·58

These ratios are obtained from means of winds from all directions irrespective of the stability of the air. He also gives percentage frequencies of v_s/v_g between certain limits ranging from 0·24 to 1·08 calculated by Jeffries from ships' observations over the North Sea for the various quadrants. These frequencies group themselves around a mode of 0·67 for the north-west quadrant and around a mode of 0·55 for the north-west and south-west quadrants. These frequencies have also been obtained without regard to stability considerations.

The majority of text books give 2/3 as the ratio of v_s/v_g over the sea. Frost (1948) quotes that Brooks has stated that a value of 0·60 is more nearly true. The values given in Table I include winds from all directions and are probably unrepresentative of values over the open sea. In comparison they may be low due to the influence of the land immediately around the anemometer, even to seaward. At low tide the area of exposed beach may be considerable and this may reduce the surface velocity of onshore winds.

Frost (1948) gives observed ratios of 0·66, 0·60 and 0·55 for onshore winds of 10-19 mi/hr, 20-29 mi/hr and 30-39 mi/hr respectively.

Frost also gives a generalised theoretical variation of the ratio v_{10m}/v_g with lapse rate over the sea for $v_g = 10$ m/sec and roughness parameter $z_0 = 1$ cm. This ratio decreases from 1·00 with very stable lapse rates to a minimum of 0·55 and then increases to 1·01 for very unstable lapse rates. However, observational evidence is not given to support this theoretical variation over the sea.

2. Observations

The ratios v_{50}/v_{2000} were obtained from ocean weather ship observations at Stations ITEM 60°N. 20°W. and JIG 53°50′N. 18°40′W. Observations of v_{50} represent wind velocities recorded by the anemometer situated on the mast at a height of approximately 50 ft above the sea. Observations of v_{2000} and of the temperature at 2000 ft were obtained from the upper air ascents by interpolation.

3. Relation between v_{50}/v_{2000} and the lapse rate

Table II gives the mean values of the ratio v_{50}/v_{2000} for each 1°F per 1000 ft class interval of lapse rate for overlapping adjacent class intervals. The overlapping values have been obtained from the smoothing formula $\frac{f_1m_1 + f_2m_2}{f_1 + f_2}, \frac{f_2m_2 + f_3m_3}{f_2 + f_3}$ *etc.*, where m_1, m_2, are the numerical values of the classes and f_1, f_2 the class frequencies.

TABLE II. Relation between mean values of $\frac{v_{50}}{v_{2000}}$ and the lapse rate over the sea

Lapse Rate °F/2000 ft	Mean $\frac{v_{50}}{v_{2000}}$	No. of cases	Mean $\frac{v_{50}}{v_{2000}}$ Overlapping adjacent values	No. of cases
− 9	73	1		
− 8	—	—		
− 7	69	1		
− 6	65	1		
− 5	63	5		
			·56	13
− 4	51	8	·60	19
− 3	67	11	·61	22
− 2	55	11	·64	27
− 1	70	16	·70	30
0	70	14	·69	38
1	69	24	·66	65
2	65	41	·66	90
3	67	49	·64	126
4	62	77	·62	129
5	63	52	·66	98
6	69	46	·71	141
7	72	95	·71	189
8	70	94	·73	239
9	75	145	·75	239
10	75	94	·75	174
11	76	80	·75	119
12	74	39	·75	52
13	78	13	·81	19
14	83	6		
15	70	1		
16	96	1		
17	—	—		
18	—	—		
19	87	1		
		Total 926		

The curve based on the smoothed values is illustrated in Fig. 1. It is seen that the ratio slowly increases from a primary minimum in inversion conditions as the magnitude of the inversion decreases. The ratio reaches a primary maximum of 0·70 when an isothermal lapse rate is approximately reached and then decreases to a secondary minimum with a lapse rate of about 2°F/1000 ft. The ratio then increases again steadily as the lapse rate increases.

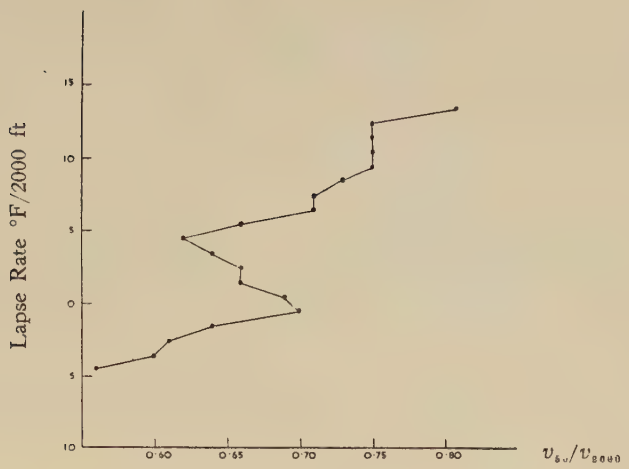

Figure 1. Variation between $\frac{v_{50}}{v_{2000}}$ and lapse rate.

It is noted that the main features of the curve agree with Frost's (1948) theoretical relationship over the sea except for inversion conditions. However, Frost's relationship is limited by the conditions of a roughness parameter of $z_0 = 1$ cm whereas the roughness in the observations illustrated by Fig. 1 may be greater than this. It is noted that Frost's relationship over land, where z_0 is much greater than over the sea, shows a decrease in the ratio as the inversion increases. Frost supports this relationship with observational evidence.

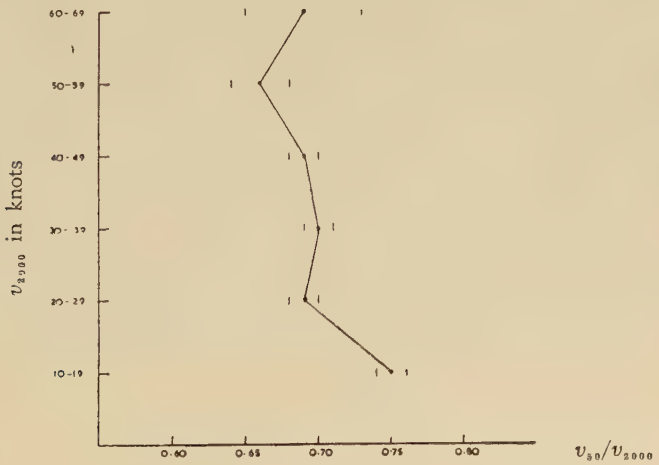

Figure 2. Variation of $\frac{v_{50}}{v_{2000}}$ with v_{2000} in knots.
Distance between short vertical lines $= 2\sigma_m$

The mean value of v_{50}/v_{2000} calculated from 710 values taken at random is 0·71 which agrees with the normally accepted value of 2/3 based on observations of surface wind referred to a height of 33 ft above the sea.

4. Relation between $\frac{v_{50}}{v_{2000}}$ and v_{2000}

Fig. 2 illustrates the variation of the ratio v_{50}/v_{2000} with v_{2000}. The standard deviation of the mean σ_m is shown on the figure for each point. They are 0·18, 0·17, 0·13, 0·13, 0·14, 0·13 respectively for each point of increasing wind velocity range.

There is a fall in the ratio as the velocity increases from the 10-19 kt range to the 20-29 kt range. The ratio then remains fairly constant for other values of v_{2000} from 20-29 kt to 60-69 kt.

Acknowledgment

I am indebted to the Director of the Meteorological Office for permission to publish this note.

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Frost, R. 1948 *Quart. J. R. Met. Soc.*, London, **74**, p. 316.
Shaw, Sir Napier 1930 *Manual of Meteorology*, Cambridge (Univ. Press), **4**, p. 95.

Meteorological observations at the Royal Observatory, Greenwich, during the year 1949

(Communicated by the Astronomer Royal)

The normal programme of meteorological observations at the Royal Observatory has been continued during 1949.

The maximum temperature of the air was 89·3° F, which was reached on 27 June. The corresponding reading of the maximum thermometer exposed on the open (Glaisher) screen was 92·0°. The temperature rose to 80° or above on 29 days, 12 of these in July and 9 in August. A reading of 84·0° recorded on 16 April during a very warm Easter was the highest April reading ever recorded at Greenwich.

The minimum temperature during the year occurred on 4 February and was 19·8°. The minimum thermometer on the Glaisher screen was stolen less than twelve hours before this minimum occurred, so that the corresponding reading on the open stand is unknown. There were 38 days on which the temperature fell to 32° or below ; 10 of these occurred in March.

The mean temperature of the year was 52·4°, which is 2·7° above the 100 years' average and the highest on record. This was the seventh consecutive warm year. Except for March and May, every month was above average. The outstanding month was September with a mean temperature of 63·7°, 6·4° above average, and only 0·1° cooler than the record September of 1865. Six other months had mean readings exceeding their average by at least 3·0°.

The maximum reading of the solar radiation thermometer was 150·0° on 14 July. The minimum temperature on the ground was 6·5° on 3 and 4 February. The difference between the highest and lowest readings of the radiation thermometers exceeded 100° on 14 days, and was greatest (112·5°) on 2 July.

The reading of the barometer at 152 ft above mean sea level ranged from 28·310 in. on 1 January to 30·668 in. on 30 January.

The total of *bright* sunshine registered in the year was 1597·8 hours, which is 164 hours above the average. This was the sunniest year since 1929 and only March, May and September were below average. February was the second sunniest on record with a total of 102 hours. The sunniest day was 3 July with 15·2 hours, representing 92 per cent of the total possible. There were 55 entirely sunless days, only 5 of which occurred during the six months April to September.

The night sky camera recorded the trace of δ Ursae Minoris for 46 per cent of the total exposure. In comparison, 36 per cent of the possible sunshine was recorded. Ten days were overcast throughout and three cloudless throughout.

The total horizontal movement of the air during the year was 89,517 miles. The greatest daily movement was 710 miles on 4 April. The greatest hourly movement was 41 miles on 9 February, and the greatest pressure of the wind, 33·0 lb/ft^2, on the same day.

The total rainfall for the year was 18·648 in. which is 5·682 in. less than the 100 years' average. Only 9·923 in. had been recorded by the end of September, whereas 5·021 in. fell in October, the only month with more than average rainfall. The largest amount on one day was 1·274 in. on 26 October. There were seven periods of ten or more consecutive days and a total of 240 days on which no measurable rain fell ; the longest period being one of 19 days, 15 June to 3 July.

Reviews

Climate through the ages. By C. E. P. Brooks. London (Ernest Benn Ltd.) 2nd Edition, revised, 1949. 8vo. Pp. 395 ; 39 figures, 31 tables. 21*s.*

Books on climatology are of two kinds. The first kind, which includes the great majority, deals with the climate of the present : it is concerned with the distribution of climate in space but not in time. The second kind is more ambitious and attempts to deal with climatic change. Now climatology, regarded as a science, differs from meteorology only in point of view and meteorology can be understood only in terms of processes, *i.e.* in terms of development. How much of meteorology should we know if we had only one synoptic chart ? It seems reasonable to suppose then that we are likely to understand climatology better if we consider climate in terms of its development. To a meteorologist such an understanding is of particular interest and importance because the factors which ultimately determine climatic changes are the same as those which determine weather. Unfortunately the study of climatic change is doubly difficult — as much labour and ingenuity is needed to discover the facts (one is tempted to copy the journalists and write " the true facts ") as to discover the explanation of these facts. Moreover the background knowledge needed for discovery is quite different from that needed for explanation — in the former case geological, in the latter meteorological. Add to these difficulties the fact that the available evidence is meagre and most often dubious and it is clear that the man who is to make sense of it must have not only wide knowledge but very considerable critical ability. Fortunately Dr. Brooks measures up to requirements and in *Climate through the ages* he has produced a work of major importance. We are most of us aware of Dr. Brooks's large output of writings on meteorological and climatological subjects. This is his *magnum opus* the work for which, first of all, he will be remembered in time to come.

Because of its comprehensiveness *Climate through the ages* is unique. Not only does it deal with all time scales from the very distant to the very recent past and with the various methods by which real or supposed evidence is obtained but a wide variety of theories (ranging from the plausible to the completely unfounded) is examined and an attempt made to get some sort of consistent picture. A great deal of compression provides plenty of " meat " for the serious student but this is no " dry as dust " textbook. Cheerfulness keeps breaking through and in Chapter XI for example we have a lyrical intermezzo of the highest quality. Good writing goes easily with good sense and *Climate through the ages* is notable for its clarity and commonsense. It is surprising how much information Dr. Brooks is able to derive from what appear to be simple empirical methods but which in fact depend for their success on broad experience, a sense of proportion and not a little " flair." Dr. Brooks rightly emphasises the importance of quantitative considerations in attempting to decide between a multitude of hypotheses and wherever possible his arguments are reinforced by numerical estimates.

Climate through the ages represents both the consolidation of previous work and the opening of a new chapter in the history of climatology. For the first time we can see, albeit still very dimly, the general nature of the major problems to be solved and can hazard plausible guesses about the solution of some of them. If Dr. Brooks's theoretical analysis is in many places quite inadequate and unsatisfying (*e.g.* regarding the general circulation of the atmosphere) we cannot blame him because theory is not yet sufficiently developed. Rather we should thank him for making so many bricks with so little straw. It may well be that in the end a more precise analysis will not give such very different results. And if in some instances a case is somewhat overstated (*e.g.* the case against " continental drift " which many would say is not such a dead letter as Dr. Brooks makes out) we have at least the problem indicated. No one has done better than Dr. Brooks. Few can hope to do as well. Get this book and read it.

E.T.E.

Dynamics of real fluids. By E. G. Richardson. London (Edward Arnold & Co.) 1950. Pp. vii 144, 96 figures. 21*s.*

The meteorologist concerning himself with the behaviour of those very real fluids, air and water, under other conditions than in his special study — a proper concern, since nature has few clear-cut boundaries and many parallels — has heretofore been largely dependent upon the two volumes of *Modern Developments in Fluid Dynamics*, edited by Goldstein. But Goldstein is essentially a work of reference, deals only with some parts of the field, and is on the whole adapted more to the needs of the theoretical than to those of the experimental physicist. Consequently the appearance of a work on fluid dynamics which is wide in scope and concerned much more with physical principles and methods of investigation than with mathematical detail is assured of a sympathetic reception by many of us.

Dr. Richardson is well known for his work on fluid flow and there are few parts of the field into which he has not adventured. We can therefore expect from him an integrated view. In the present volume he deals in turn with the classical methods for potential and viscous flow, boundary layer flow in fluids of small viscosity such as air and water — a few pages here are given to atmospheric turbulence, compressible fluids, heat transfer phenomena, disperse systems (aerosols, *etc.*), and elastic fluids (gels). All but the last and compressible-fluid flow, that is flow at near-sonic speeds, are of interest to the meteorologist, and the book might therefore have provided him with a good " point of departure " for a number of his own characteristic problems. That it hardly does so is to be regretted and we doubt whether this is in any way due to excessive demands from the meteorologist.

There are two main reasons for our disappointment. First, this is a book of less than 150 pages and how, one may ask, can any author hope to be so comprehensive in such small space, and yet provide a coherent picture of the parts or the whole ? With about twice the space at his disposal he might well have realised his aims (nowhere stated, it is true, but readily enough inferred) and given the reader a yet quite manageable volume. Disappointment springs secondly from the frequent use of loose and uncertain, and sometimes even false, constructions, the result in part no doubt of the compression. For example, steady flow is in one place equated with $dU/dt = 0$ instead of $\partial U/\partial t = 0$, the velocity potential is stated to be the line integral of the circulation, the concepts of *vortex* and *vorticity* are introduced without definition, as are *rotational* and *irrotational*, and, in a different sort, formulae are stated without making it clear whether they should be obvious to the reader or are the result of analysis not reproduced. In consequence the reader is liable to be confused or misled unless he already brings to the text a moderately good knowledge of basic hydrodynamics. Without it he will find the book much stiffer reading than it need have been or than the author presumably intended.

But this criticism having been made, the book has value, and the meteorologist will undoubtedly find much relevant material within its covers that is hardly to be found in any other one English text. We give a few instances : the theoretical work of Tollmien and Schlichting and the experimental work of Schubauer and Skramstad on unstable disturbances in a boundary layer ; the dissipation of heat from cylinders and the evaporation from or condensation on to spheres ; the laws of the break-up of liquid filaments and drops and of the fall of drops ; the mechanics of turbulent suspensions such as dust in air ; and, for the applied meteorologist, the principles, as far as they have been ascertained, of erosion.

The book is well illustrated and referenced but its price, even today, seems rather high for a volume of such moderate bulk. P.A.S.

The measurement of air flow. By E. Ower. London. (Chapman & Hall, Ltd.) 1949. 8vo. Pp. 293 : 101 figures. 30*s*.

The measurement of air flow, the present edition of which is a third and revised form, is written specifically for ventilation engineers and students. The emphasis is accordingly on the appropriate practical aspects and only passing reference to meteorological applications is made in the short introductory chapter. However, the principles of the measurement of air flow are the same, though the practical difficulties may differ, whether the air is moving through a pipe or over the earth's surface. Thus apart from certain special technical points much of the book is likely to be of interest to meteorologists in general and of practical value to those meteorologists dealing with problems of anemometer design and calibration.

Three chapters are devoted to considerations of the principle, design and use of the standard of air-flow measurement — the pitot-static tube, which has special meteorological interest in view of its appearance in modified form as the historic Dines pressure-tube anemometer. The vane anemometer, the development and study of which owe much to the author's own efforts, occupies a substantial chapter. At one time this instrument was the best available for accurate explorations of the wind velocity distribution near the ground, but in this field it has been superseded by the more suitable sensitive cup anemometer. Methods depending on the cooling power of the air, mainly the well known hot-wire anemometer, are described in another chapter. The latter instrument is rarely used by the meteorologist, and then only in special studies of speed fluctuations.

The chapter on pipe flow — a new chapter stimulated by recent advances in the subject — is of interest in view of the many successful applications of the associated boundary layer concepts to problems of flow near the surface of the earth.

Remaining chapters deal usefully with the special manometers demanded by pitot-static tube technique, miscellaneous methods of flow measurement and some practical examples. F.P.

Proceedings at meetings of the Society

Ordinary Meeting, 22 February 1950

The election of the following candidates was confirmed :—

Fellows

Bell, James, M.R.S.T., F.R.G.S., 18 St. Catherine's Grove, Lincoln.
Blundell, Denis Ewart, 2 Hill View Road, Orpington, Kent.
Bushby, Frederick Henry, B.Sc., A.R.C.S., 20 East Avenue, Wallington, Surrey.
Chilton, Donovan, B.Sc., A.Inst.P., Science Museum, Exhibition Road, London, S.W.7.
Crawford, Iain M., B.Sc., 12 Golf Road, Burnside, Rutherglen, Lanarkshire.
Cull, Desmond George, B.Sc., A.R.C.Sc., Clarence House, Llandovery, Carmarthenshire.
Morrison, Robert Bruce, 127 Ankerdine Crescent, Shooters Hill, London, S.E.18.
Newton, Thomas Ronald, A.R.Ae.S., M.I.N., 60 Cunningham Park, Harrow, Middlesex.
Pearce, Robert Penrose, 12 Woodlands Road, Barnes, S.W.13.
Powell, David Albert, 47 Empress Avenue, Hamilton, Ontario.
Roe, Dennis, " Fern Bank," Sutton-in-Craven, Nr. Keighley, Yorks.
Taylor, Edward Frank, 77 Croydon Road, London, S.E.20.
Verge, Robert William, 115-7th Avenue, N.W., Calgary, Alberta.

Associates

Andres, Otto. J., Aviation Forecast Office, Fort Churchill, M.P.O. 1015, Via Winnipeg, Manitoba.
Blackman, Sol Louis, Aviation Forecast Office, R.C.A.F. Station, Greenwood, Nova Scotia.
Clark, Wm. Gilmour, 330 Cameron Road, RR.8, New Westminster, B.C.
Cooper, Cedric Bond, 3530 West First Avenue, Vancouver, B.C.
Cooper, Wm. E. H., Main Met. Office, Goose Airport, Newfoundland.
Creswick, Wm. S., Aviation Forecast Office, R.C.A.F. Station, St. Hubert, P.Q.
Currie, Donald B., 11638-87th Street, Edmonton, Alberta.
Dennis, Arnett Stanley, Aviation Forecast Office, R.C.A.F. Station, Greenwood, Nova Scotia.
Dickison, Robert B. B., Aviation Forecast Office, R.C.A.F. Station, Chatham, New Brunswick.
Emslie, John Hugh, 12408-112 Avenue, Edmonton, Alberta.
Fryers, W. R., Ste. 5 Bruce Apartments, Corner Victoria and St. John, Regina, Saskatchewan.
Gillis, John J., 10929-123 Street, Edmonton, Alberta.
Glenesk, Norman A., District Aviation Forecast Office, P.O. Box 283, Moncton, N.B.
Harvey, Wm. Franklin, Aviation Forecast Office, R.C.A.F. Station, Trenton, Ontario.
Holbrook, Rodney Kitt, Met. Office, R.C.N. Air Station, Dartmouth, N.S.
Hone, Antoine L., 5717 Notre Dame de Grace Avenue, Montreal, Quebec.
Hyde, Eric Norman, 3930 West 23rd Avenue, Vancouver, B.C.
Johnson, Glen Lee, 39 South Street, Halifax, Nova Scotia.
Knight, Paige Anderson, District Aviation Forecast Office, Montreal A/P, Dorval, Quebec.
Kowal, Paul R., Aviation Forecast Office, R.C.A.F. Station, Trenton, Ontario.
Lauder, John Richard, 10446-133 Street, Edmonton, Alberta.
Lee, Roy B., Box 22, Gander, Newfoundland.
Mackie, Wm. Harvie, 130 Wellington Crescent, Sea Island, Vancouver, B.C.
Macleod, Malcolm G., Aviation Forecast Office, R.C.A.F. Station, Trenton, Ontario.

Mahaffy, F. R., 2037 Queen Street, Regina, Saskatchewan.
McQuarrie, Allan F., Aviation Forecast Office, Municipal Airport, Calgary, Alberta.
Millar, Lloyd T., Aviation Forecast Office, R.C.A.F. Station, Greenwood, N.S.
O'Brien, Ralph Harvey, 59 Queen Street, Dartmouth, Nova Scotia.
Oddy, Leonard Cameron, Aviation Forecast Office, Fort Churchill, M.P.O. 1015, Via Winnipeg, Manitoba.
Pambrun, Robert Charles, Aviation Forecast Office, R.C.A.F. Station, Trenton, Ontario.
Pankewich, A., 12419-67 Street, Edmonton, Alberta.
Pattison, Robert F., Met. Office, M.P.O. 1006, Rivers, Manitoba.
Peach, James A., Bay Roberts, Newfoundland.
Rabnett, Jerome Oscar, 985-6th Avenue E., Owen Sound, Ontario.
Ripenburg, Ralph Herbert, Aviation Forecast Office, R.C.A.F. Station, Trenton, Ontario.
Rogalsky, John, Aviation Forecast Office, Fort Churchill, M.P.O. 1015, Via Winnipeg, Manitoba.
Sawchuk, Walter, 2033 Hogan Street, Montreal 24, Quebec.
Schwentke, Otto, 1801 Quebec Street, Regina, Saskatchewan.
Smith, Donald K., Aviation Forecast Office, H.M.C.S. Shearwater, Dartmouth, Nova Scotia.
Stark, Robert A., 20 Strathmore Blvd., Strathmore, Quebec.

Student Associate

Bell, Basil Edward, 11 Cambridge Park Court, Twickenham, Middx.

The following paper was read :

" The bright band — a phenomenon associated with radar echoes from falling rain."

By J. E. N. Hooper, B.Sc., and A. A. Kippax, B.Sc. (see *Q.J.*, Vol. 76, p. 125).

This was followed by a discussion on " Radar echoes " opened by Messrs. R. F. Jones, J. E. N. Hooper and A. A. Kippax.

No Ordinary Meeting was held in March owing to the proximity of the Centenary Celebrations, 28 March — 3 April, a full account of which will be given in the Special Centenary Issue of the Journal.

Ordinary Meeting, 19 April 1950

The election of the following candidates was confirmed :—

Fellows

Bruce, Charles Edward Rhodes, 5 Wadsworth Road, Greenford, Middx.
Champniss, Gerald Arthur, 96 Newbury Gardens, Stoneleigh, Ewell, Surrey.
Hare, Alexander Edward, " Addiscombe," Cranborne Road, Bournemouth.
Langdon, Sqn. Ldr. Arthur, 19 Mount Pleasant Road, New Malden, Surrey.
Peter, Norman, 43 Morpeth Avenue, South Shields, Co. Durham.
Radcliffe, Harold Vernon, 33 Ramsden Road, Rotherham, Yorks.
Rawlins, Donald Frank, " Sainfoin," 70 Pier Avenue, Herne Bay, Kent.
Rutledge, Col. T. Lloyd Forster, Gidleigh, Bungendore, N.S.W., Australia.
Spencer, Cecil Walter, B.Sc., 150 Broom Road, Teddington, Middx.

Student Associates

Chuter, Ian Henry, 64 West Way, Shirley, Croydon, Surrey.
Ovenden, John David, 27 Cardinal Crescent, New Malden, Surrey.
Woodburne, Alison Vera, Flat 2, 19 Holland Park, W.11.
Wood, Brian James, Dormer Cottage, Church Road, Worcester Park, Surrey.

Professor P. M. Blackett, F.R.S., delivered the Symons Memorial Lecture, his subject being " Theories of the origin of cosmic rays."

Ordinary Meeting, 17 May 1950

The election of the following candidates was confirmed :—

Fellows

Bilham, Ernest George, B.Sc., D.I.C., Meteorological Office, Dunstable, Beds.
Callow, Raymond Leslie, M.A., B.Sc., B.D., 31 Elphinstone Road, Hastings, Sussex.
Chaplin, James Harvey, 36 Courtfield Avenue, Harrow-on-the-Hill, Middlesex.
Davies, David Rowland, 104 Furniss Avenue, Dore, Sheffield.
Dwyer, Walter A., Central Meteorological Bureau, Melbourne, Australia.
Hogan, John, B.Sc., Commonwealth Meteorology Bureau, Observatory Park, Sydney, N.S.W., Australia.
Holgate, Harold Theodore Dixon, 3 Ash Mount Road, Grange-over-Sands, Lancs.
Joesbury, John Douglas, 171 Bolden Road, Harborne, Birmingham.
Maddock, Alfred George, A.I.Struct.E., 131 Stoke Road, Guildford, Surrey.
Martin, Geoffrey Noel, The Observatory, Crowborough, Sussex.
Pack, Instr. Captain, Stanley Walter Crowther, M.Sc., M.I.E.E., R.N., Staff Meteorological Officer, Office of Flag Officer (Air), Lee-on-Solent, Hants.
Roe, Peter E. A., 21 Glebe Way, Hornchurch, Essex.
Sumner, Eric James, 254 Waterloo Road, Cobridge, Stoke-on-Trent, Staffs.
Viner, Flt. Lt. Derek Reginald Watson, c/o 87 Stanley Hill Avenue, Amersham, Bucks.
Weir, James Norman McLeod, 169 Belle Vue Road, Southbourne, Hants.
Worswick, Ronald Francis Kevin, 2 Freckleton Avenue, Chorlton-cum-Hardy, Manchester, Lancs.

Foreign Members

Daniel, Cerge, National Meteorological Service, Air Ministry, Athens, Greece.
Fea, Professor Giorgio, Via Salarie 195, Rome, Italy.
Lowell, Sherman C., Ph.D., London Office for Naval Research, U.S.A., 34 Grosvenor Square, W.1.
Montgomery, Professor Raymond B., Sc.D., Brown University, Providence 12, Rhode Island, U.S.A.
Panofsky, Professor Hans A., 318a Plaza Road, Fairlawn, New York, U.S.A.
Riehl, Professor Herbert, University of Chicago, Illinois, U.S.A.
Schaefer, Vincent J., Sc.D., R.D. 3 Schermerhorn Road, Schenectady, New York, U.S.A.
Simon, Albert, 13 Bustan Street, Cairo, Egypt.
Van Mieghem, Professor Jacques, M.L.G., 52 Rue des Glaieuls, Uccle, (Brussels).
Vourlakis, Gp. Capt. Evangelos, Director of Greek National Meteorological Service, Athens, Greece.
Willett, Professor Hurd Curtis, B.S., Ph.D., Room 24-606, M.I.T., Cambridge, Mass., U.S.A.

ASSOCIATE

KNOX, JOHN LEWIS, 8 Islington Avenue South, Toronto 18, Ontario, Canada.

STUDENT ASSOCIATES

NEWELL, REGINALD EDWARD, 6 Orchard Street, Peterborough, Northants.
RENOUF, STANLEY WILLIAM, 6 Macklin House, Shackleton Close, Forest Hill, S.E.23.

The following paper was read :

"Variations in thunderstorm severity in Great Britain."
By J. S. FORREST, M.A., D.Sc.

This was followed by a talk on :—

"Recent work of the Thunderstorm Census Organisation" by its Director, Mr. S. MORRIS BOWER.

Several members and visitors took part in the discussion which followed.

Recent additions to library

J. R. ASHWORTH, *Ultra-violet daylight rays in the solar cycle.* The University Press of Liverpool. Pp. 73. 1949.

W. G. V. BALCHIN and A. W. RICHARDS, *Climatic and weather exercises.* Harrap and Co., Ltd. Pp. 79. 1949.

M. BORN and H. S. GREEN, *A general kinetic theory of liquids.* Cambridge University Press. Pp. 98. 1949.

C. R. BURGESS, *Meteorology for seamen.* Brown, Son and Ferguson, Ltd. Pp. 252. 1950.

S. CHANDRASEKHER, *Radiative transfer.* Oxford University Press. Pp. 408. 1950.

THE FARADAY SOCIETY, *Crystal growth.* Guerney and Jackson. Pp. 336. 1949.

M. FISHENDEN, *An introduction to heat transfer.* Oxford University Press. Pp. 212. 1950.

E. E. FOSTER, *Rainfall and runoff.* Macmillan and Co. Pp. 487. 1949.

F. HOYLE, *Some recent researches in solar physics.* Cambridge University Press. Pp. 134. 1949.

H. JEFFREYS, *Earthquakes and mountains.* Methuen and Co. Pp. 191. Revised edition 1950.

I. LANGMUIR, *Phenomena, atoms and molecules.* The Philosophical Library, New York. Pp. 436. 1950.

L. DON LEET, *Earth waves.* Chapman and Hall Pp. 122. 1950.

LEO LYSGAARD, *Lufthav, Vejr og Klima.* Copenhagen. Gyldendalske Boghandel Nordiske Forlag. Pp. 232. 1943. (Presented to the Society by the Author.)
Recent Climatic Fluctuations. Copenhagen. I Kommission Hos H. Hagerups Forlag. Pp. 214. 1949. (Presented to the Society by the Author.)

L. MILNE-THOMSON, *Theoretical hydrodynamics.* Macmillan and Co. Pp. 600. 2nd edition 1949.

E. OWER, *The measurement of air flow.* Chapman and Hall. Pp. 293. 3rd edition 1949.

E. G. RICHARDSON, *The dynamics of real fluids.* Edward Arnold. Pp. 144. 1950.

O. G. SUTTON, *The science of flight.* Penguin Books. Pp. 208. 1949.

M. V. WILKES, *Oscillations of the earth's atmosphere.* Cambridge University Press. Pp. 76. 1949.

Obituary

HUGH ROBERT MILL, D.Sc., LL.D., F.R.S.E.

One of the last remaining links with mid-Victorian meteorology was broken on 5 April 1950 by the peaceful death in ripe old age, at his home near East Grinstead, of Hugh Robert Mill. Heroes of classical times, we are told, used to ask as a supreme boon of the gods that they might be allowed to die in their hour of greatest triumph. If Mill ever entertained such a wish it may be said to have been granted. The closing weeks of his life were made bright for him by what he regarded as the highest distinction that had come his way — the award to himself of the Medal and Prize newly founded by the Royal Meteorological Society in his honour and bearing his own name. (See Plate I). He had learnt late in 1949 that his important contributions to knowledge of the rainfall of the British Isles had been thus recognised, but public announcement of the award was not made until the centenary commemoration meeting of the Society on 3 April 1950, two days before his death. Hence he was unable to receive all the congratulatory messages which friends and former colleagues wished to send him on the latest of the many distinctions accorded to him as geographer and meteorologist.

Mill was born in 1861 at Thurso, Caithness, the son of Dr. James Mill, a physician who had begun his medical practice as surgeon in a whaler to the Greenland seas. The boy's school education was seriously interrupted by ill health, but he made up for this by intensive work at home. He became, in his own words, " an omnivorous reader, finding interest in every book, magazine and newspaper." In 1876 the Mill family migrated from Thurso to Edinburgh, and there young Hugh Robert provided himself with a rough set of meteorological instruments, including a home-made rain-gauge which he subsequently confessed to have been of such crudity that he shuddered to think of it. With this equipment and a copy of Alexander Buchan's *Handy Book of Meteorology* he embarked on his studies of our science that were eventually to prove so fruitful.

In 1880 Mill entered the University of Edinburgh, taking chemistry and physics as his subjects for a degree. Possession of a pocket spectroscope for use in chemical experiments resulted in his paying special attention to the absorption lines produced by atmospheric water vapour in the spectrum of sunlight reflected from the sky as related to approaching weather changes. On these youthful investigations was based his first printed contribution to scientific literature — a short paper read before the Royal Society of Edinburgh on 9 January 1883 at the instance of Prof. P. G. Tait. A little later, by request of Adam Hilger, the maker of his spectroscope, Mill wrote a pamphlet entitled " The rainband, how to observe it, and what to expect from it."

In 1884 Mill, as a young graduate, was appointed chemist and physicist to the Scottish Marine Station, then newly established by John Murray (of Challenger fame) at Granton, on the Firth of Forth. His duties there were mainly concerned with collecting and analysing specimens of sea water and observing temperatures in the estuaries and lochs of Scotland ; but he was also put in charge of a regular second-order meteorological station on shore under Buchan's supervision. While so engaged he came into association with John Aitken's pioneer work on the part played by nuclei in the formation of rain and also helped Prof. Chrystal with anemometrical experiments. After the termination of this appointment Mill became lecturer on physiography in the Heriot Watt College, Edinburgh. In that capacity he was called upon to write elementary geographical and physiographical text-books, one of which, *The Realm of Nature*, achieved considerable popularity. It was his constant aim in these books, as well as in the lectures on the same subjects which he gave under the University Extension scheme, to place geography on a sound scientific foundation.

In 1892 Dr. Mill, as he was by then, having received the Edinburgh D.Sc. degree six years earlier, assisted the Royal Geographical Society in equipping for scientific work the sailing ships Balaena and Active, which voyaged from Dundee to explore the whaling potentialities of the Antarctic. Late in the same year he accepted the post of librarian to the Royal Geographical Society. His many activities in that office included the preparation of a subject catalogue of geographical literature and of a new authors' catalogue of the entire library ; he also took a hand in editing the *Geographical Journal*. During the next few years he nevertheless found time to construct and publish maps of the beds of the largest English lakes, to edit, with J. Scott Keltie, the *Report of the Sixth International Geographical Congress* (1896), and to initiate a scheme for relating agricultural statistics to topography, rainfall, and other meteorological elements. A comprehensive survey of the British Isles on such lines was among his principal ambitions at this period ; to set an example for the project he made a special study of south-west Sussex, the results of which were embodied in a paper read before the Royal Geographical Society on 5 February 1900. G. J. Symons was present to take part in the discussion. Later that year Mill's eminence as a geographer was recognised by the award to him of the honorary LL.D. degree of St. Andrews University.

The first day of the twentieth century was to prove one of the most important in Mill's career. After the death of G. J. Symons on 10 March 1900, his successor as head of the British Rainfall Organization, H. Sowerby Wallis, soon found himself in need of a partner and invited Mill to share with him the conduct of the work. This gave the geographer a long-sought opportunity to study in detail the control of rainfall distribution by the forms of the land. He therefore accepted Sowerby Wallis's invitation, resigned his post with the Royal Geographical Society, and on 1 January 1901 took up his duties at Camden Square. A professional meteorologist thenceforward, he threw himself with characteristic energy into his new activities, following initially the main lines of investigation marked out by Symons but soon branching off along byways which Symons had never explored. As a trained scientist Mill was not satisfied with the mere collection and discussion of observations ; they appealed to him chiefly by their challenge for explanation. To this he devoted a great deal of his time and energy for close on twenty years. It appears to have been Mill who first noticed that many cyclonic rains seemingly have their field controlled by the direction and degree of curvature of the track followed by the responsible low-pressure centre, and that excessive downpours are apt to occur near the point where a slow-moving depression suddenly changes course.

In the August of 1903 failing health caused Sowerby Wallis to end his long association with the British Rainfall Organization ; he had joined Symons in 1872 and had acted as assistant editor of its publications since 1890. On his retirement, Mill purchased the accumulated records and other property of the Organization and became solely responsible for the conduct of its work. As time went on he introduced numerous improvements into *British Rainfall* and *Symons's Meteorological Magazine*, both of which found increasing scientific appeal through his skilled guidance, clarity of exposition and superlative accuracy of mind.

Tribute to his abilities as a meteorologist had been paid by this Society soon after he obtained fellowship in 1893 ; in 1895 he was elected to the Council, and in 1902 to honorary secretaryship, a post which he held until proceeding to presidency in 1907. Subsequently, in 1918, he received the Symons Memorial Medal, the Society's highest award. It was during Mill's term of office as president that the Meteorological Dining Club was first formed. He proposed that it should be called " The Isobars," to stress the democratic principle that all members carried equal weight regardless of their status in the Society. The Council decided by a majority vote not to adopt this ingenious name. They were, however, glad to accept from Dr. Mill as an inauguratory gift the silver model of a Snowdon rain-gauge which is still used to contain the cigarettes passed round at the Club dinners and is inscribed with the names of the successive presidents from 1907 onwards.

Realising the vital importance of rainfall statistics to the welfare of a modern community, Mill felt that maintenance of the British Rainfall Organization's observational work and preservation of bygone records ought not to depend on the frail thread of one man's life. Accordingly, in 1910, he presented the entire body of data collected by Symons, Sowerby Wallis and himself, together with the remainder of the Organization's property, to be held in trust for the nation — a notable act of patriotic self-abnegation. Despite precarious health, repeated threats to his eyesight, and a serious accident which incapacitated him for some months, he acted as chairman of the trustees and remained at the helm of the Organization until it was absorbed by the Meteorological Office in 1919.

Mill's devotion to geography and oceanography was not lessened by his preoccupation with meteorology after the turn of the century. In 1901 he presided over Section E (Geography) of the British Association, which he had previously served as recorder. From 1901 to 1909 he was British representative on the International Council for the Study of the Sea. He edited *International Geography* in 1911, and in 1915 was awarded the Victoria Research Medal of the Royal Geographical Society. From 1927 to 1931 he was vice-president of that body, and for the celebration of its centenary, which fell during his tenure of office, he prepared *The Record of the Royal Geographical Society, 1830-1930*. He was Gold Medallist of the Royal Scottish Geographical Society in 1923 ; in 1932 he became president of the Geographical Association. Mill was rainfall expert to the Metropolitan Water Board from 1906 to 1919. and from 1918 to 1921 served as member of the Board of Trade committee on water power of the British Isles. Acquainted with nearly all the leading meteorologists of the late Victorian and subsequent eras, he enjoyed the personal friendship of many of them, both at home and abroad.

Although he never entered high latitudes himself, Mill was an authority of world-wide repute on Polar exploration. His enthusiasm for this subject dated from his association with Sir John Murray at the Challenger Office in Edinburgh. As librarian of the Royal Geographical Society he took a prominent part in the preparation of the *Antarctic Manual* for the Discovery expedition in 1901-1904. Thenceforward he made it his special care to keep abreast of all Polar work. Before long he came to be regarded as a kind of international repository for information on arctic and antarctic research.

Among the famous explorers who sought his counsel were Amundsen, Byrd, Mawson, Nansen, Scott, Shackleton, Stein and Sven Hedin. It became customary for a pilgrimage to be made to Mill's home after the conclusion of any Polar venture in order that his unique storehouse of data might be brought up to date by addition of the most recent records at first hand.

Through their common bond of love for poetry Mill formed a lasting friendship with Shackleton, of whose life and achievements he published an admirable account in 1923. His other writings on arctic and antarctic exploration include *The Siege of the South Pole* (1905) — an enthralling tale of scientific adventure. In 1930 he served as chairman to the board of directors of the Scott Polar Research Institute. Numerous foreign societies were proud to count him among their honorary members. In 1932 Norwegian appreciation of his labours for science and Polar exploration was signalised by his appointment as a Commander of the Order of St. Olav. Three years earlier he had received the Cullum Medal of the American Geographical Society.

Mill was blessed with the gifts of high literary ability, fluent oratory, and a keen sense of humour. A talk on rainfall which he broadcast nearly a quarter of a century ago is still remembered with pleasure by some of those who had the good fortune to hear it. (see Vol. 53 of this *Journal* (pp. 86-89). An annual treat to which his friends and chosen disciples always looked forward were the meteorological Christmas cards that he had printed from his own design when he directed the British Rainfall Organization. He bore the affliction of failing eyesight with fortitude, and even after almost total darkness had descended on him during the last months he somehow contrived to conduct part of his correspondence with pen and ink. Two happily phrased and tolerably legible letters in his own hand came to the writer of this appreciation after the news of the award of the Hugh Robert Mill Medal had been conveyed to him late in 1949, by which time he was generally regarded as a blind man.

At the memorial service for Dr. Mill, held at the Dormans Land parish church, near Lingfield, Surrey, on 15 April 1950, the Royal Meteorological Society was represented by its president, Sir Robert Watson-Watt, in company with Wing Commander R. M. Poulter, honorary secretary, Mr. L. C. W. Bonacina, vice-president, and Mr. C. J. P. Cave, former president.

E.L.H.

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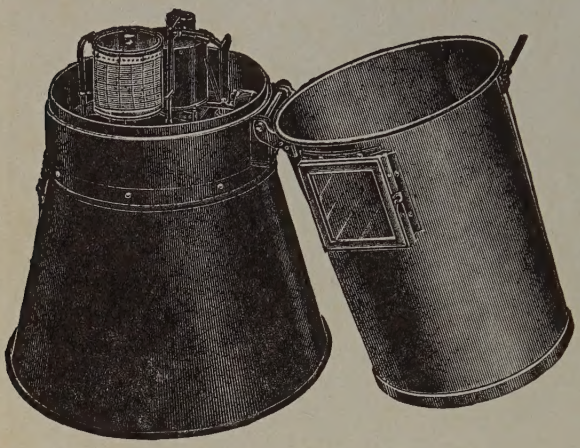

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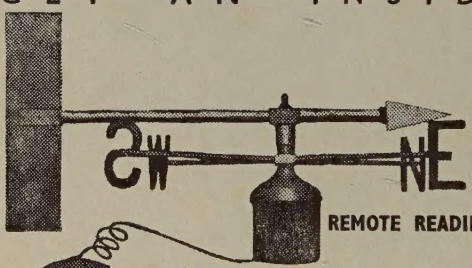

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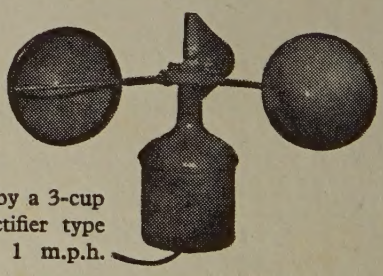

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